“PERFORMANCE ANALYSIS OF AIRCRAFT PITCH MODEL USING ROUTH ARRAY AND PADE APPROXIMATION TECHNIQUES”

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LUCKNOW

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CANDIDATE'S DECLARATION

I hereby declare that the work, which is being presented in the dissertation entitled**"**" in partial fulfillment for the award of degree of "**Master of Technology**" in Department of Electrical Engineering with Specialization in **Power System & control "PERFORMANCE ANALYSIS OF AIRCRAFT PITCH MODEL USING ROUTH ARRAY AND PADE APPROXIMATION TECHNIQUES"** and submitted to the **Department of Electrical Engineering, Babu Banarasi Das University** is a record of my own investigations under the guidance of **Prof. Akash Varshney, Babu Banarasi Das University, Lucknow.**

I have not submitted the matter presented in this dissertation anywhere for the award of any other degree.

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CERTIFICATE

It is certified that the work contained in this thesis entitled **"Model Order Reduction of High Order System Using Intelligent Techniques",** by **Priya Dhar** (Roll no.1170450004), for the award of **Master of technology** in **Electrical Engineering (Power System and Control)** from Babu Banarasi Das University has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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ABSTRACT

Model order reduction is a technique to reduce the computational complexity of the system. It reduces the order of the system which is very challenging areas of control system. Here using the mathematical models of high order dynamics system which are classified in frequency domain and time domain. In this thesis, compare the results of routh array method and Pade Approximation method with the result of original system to develop the new techniques and make simple design of model.These methods are use for finding reduced order models (ROM) of high order system.This analysis is very simple given better response when compared with another reduction method and original system.Here focusing on both theoretical and computational aspects.

The main aim of the research work is to gradually understand and evaluate the importance as well as the advantages of both the conventional and modern model order reduction methods and thus evolve the new techniques/methods to improve upon the recent as well as established methods of design. The main focus of the work is to design and develop the new methods in frequency domain both for continuous single input single output systems and multi input & multi output systems.

The modelling of a higher order system is one of the most important subjects in engineering and sciences. A model is often too complicated to be used in real life problems. It is an un-debated conclusion that, the development of mathematical model of physical system made it feasible to analyze and design. So the procedures based on the physical considerations or mathematical models are used to achieve simpler models than the original one. Whenever a physical system is represented by a mathematical model it may a transfer function of very high order.

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PRIYA DHAR

CONTENTS

LIST OF FIGURES

FIGURE NO. **DESCRIPTION OF FIGURE** **PAGE NO.**

TABLES

TABLE NO. **DESCRIPTION OF TABLE** **PAGE NO.**

LIST OF SYMBOLS

SYMBOL	DISCRIPTION
P	Instability load
E	Young modulus
I	Cross-section moment of inertia
λ	Reference length
α	Angle of attack
q	Pitch rate
ɵ	Pitch angle
δ	Elevator deflection ang
ρ	Density of air
S	Platform area of the wing
ē	Average chord length
m	Mass of the aircraft
γ	Flight path angle
$\alpha_0 L$	Zero-lift angle of attack
$CL\alpha$	Lift-curve slope of the wing
U	Equilibrium flight speed
C_T	Coefficient of thrust
C_D	Coefficient of drag
C_L	Coefficient of lift
C_W	Coefficient of weight
C_M	Coefficient of pitch moment
I_{yy}	Normalized moment of intertia

CHAPTER 1

INTRODUCTION

1.1 MODEL ORDER REDUCTION

In present day technology, there are a large number of problems which are highly complex and large in dimension. Physical systems such as aircraft, chemical plants, electrical power system networks, urban traffic networks, digital communication networks, economic systems and control system can be described mathematically by state space models or by transfer function models.

In the most practical situations of control system, high order model for the system is obtained from theoretical considerations. The high order model posses so many problems in the analysis and design. For example it takes more computational time in the analysis and design, and large complicated hardware requirement etc. It is therefore, desirable to obtain a simplified/reduced model which retains the important properties of high order system.

The modelling of complex dynamic system is one of the most important subjects in engineering. Moreover, a model is often too complicated to be used in real problems, so approximation procedures based on physical considerations or using mathematical approaches must be used to achieve simpler models than the original one.

1.2 NEED FOR MODEL ORDER REDUCTION

The approximation of linear system plays an important role in many engineering problems, especially in control system design, where an engineer is faced with controlling a physical system for which an analytical model is represented by a high order linear system.

The main objective for obtaining the reduced order model is to have a better understanding of the system, to reduce computational and hardware complexity, to make feasible designs and to obtain simple control laws. Thus among several reasons for reducing the order of a system, a few are as follows:

(a) To simplify the understanding of the system.

(b) To reduce the computational complexity for analyzing the system.

(c) To economize in terms of hardware while realizing or designing the system.

(d) To reduce the computational time while applying the models.

1.2.1 Motivations for Simplifications

The motivation for deriving simplified models may be summarized as follows

1. It simplifies the description and the analysis of the system.

2. It simplifies the computer simulation of the system.

3. It facilitates the controller design problems and yields controllers with simpler structures.

4. It reduces the computational effort in the analysis and design of control systems.

1.3 PROBLEM DEFINITION

The model of linear time invariant physical system is obtained in the form of high order differential equations by physical laws. These differential equations may be expressed in the form of a set of first order differential equations which is called a state space model and is given by (A, B, C). If the Laplace transform of high order is taken with zero initial conditions then original system may be expressed in the form of high order transfer function G(s) which is the ratio of Laplace transform of output variable and input variable. The objective of proposed research work is to obtain reduced order model either in state space form (Ar, Br, and Cr) or in the transfer function form R(s). The reduced order model will be utilized for the design of controller.

1.3.1 Frequency Domain

Let the transfer function of high order original system of the order 'n' be

$$G(s) = \frac{N(s)}{D(s)} = \frac{a_0+a_1s+a_2s^2+\cdots+a_{n-1}s^{n-1}}{b_0+b_1s+b_2s^2+\cdots+b_ns^n} \qquad \text{........ (1)}$$

Where;, $0\leq i\leq n-1$ and $0\leq i\leq n$ are known scalar constants.

$$R(s) = \frac{N(s)}{D(s)} = \frac{c_0+c_1s+c_2s^2+\cdots+c_{k-1}s^{k-1}}{d_0+d_1s+d_2s^2+\cdots+d_ks^k} \qquad \text{......... (2)}$$

Where; $0\leq i\leq k-1$ and $0\leq i\leq k$ are known scalar constants.

The objective is to realize the k^{th} order reduced model in the form of (2) from the original system (1) such that it retains the important features of the original high order system.

1.3.2 Time Domain

In time domain, the system can be described by the following state space equations

Original System

$$\dot{x} = Ax(t) + Bu(t)$$

$$y = Cx(t) + Du(t)$$

Where,

$$x(t) = n \times 1 \ \ State\ Vector$$

$$u(t) = p \times 1 \ \ Input\ Vector$$

$$A = n \times n \ \ System\ Matrix$$

$$B = n \times p \ \ Input\ Matrix$$

$$C = m \times n \ \ Output\ Matrix$$

$$D = m \times p \ \ Transmission\ Matrix$$

$$Y(t) = m \times 1 \ \ Output\ Vector$$

Reduced order system $(k < n)$

$$\dot{x}_k = A_k X_k(t) + B_k u(t)$$

$$y_k = C_k X_k(t) + D_k u(t)$$

Where,

$$x_k = k \times 1 \ \ State\ Matrix$$

$$A_k = k \times k \ \ system\ Matrix$$

$$B_k = k \times p \ \ Input\ Matrix$$

$$C_k = m \times k \ \ output\ Matrix$$

$$D_k = m \times p \ \ Transmission\ Matrix$$

$$Y_k = m \times 1 \ \ Vector\ Matrix\ of\ Reduced\ Model$$

1.4 OBJECTIVE OF THE THESIS

In the course of this period of study and research work, the main aim of the research work is to gradually understand and evaluate the importance as well as the advantages of both the conventional and modern model order reduction methods and thus evolve the new techniques/methods to improve upon the recent as well as established methods of design.

The main focus of the work is to design and develop the new methods in frequency domain both for continuous single input single output systems and multi input & multi output systems.

Finally, the reduction methods have been developed

1. The application of Routh Array and Pade Approximation method.
2. Compare the application of Routh Array and Pade Approximation methods.

1.5 SCOPE OF THE PROJECT

In this thesis a new model reduction method has been proposed for the higher order LTI systems, which is very simple compared to other existing popular methods. It is also clear that the proposed method also gives better approximation compared to the some complicated optimization based model reduction methods. In this method, the denominator polynomial of the reduced model is obtained by model method and the numerator polynomial is calculated by a simple mathematical algorithm discussed in literature. The reduced models yielded by the proposed technique are guaranteed to be stable given that the original models are stable. The reduced models also guaranteed the retention of dominant poles, steady state and transient responses of the original models.

1.6 OUTLINE OF THE THESIS:

Chapter 1 [NTRODUCTION] gives an introduction to the concept of model order reduction and the objectives of study. Various methods for model order reduction over the years by various scientists and researchers are reviewed briefly.

Chapter 2 [LITERATURE SURVEY] gives a description on classical control method and modern control method of model order reduction, and includes notes about the classification of frequency domain and time domain.

Chapter 3 [AIRCRAFT PITCH MODEL] introduction of aircraft pitch axes, Principal axes, Pitch control and stability theory, Description of aircraft flight control, Aircraft flight control surfaces, Aircraft primary control surfaces, Aircraft secondary control surfaces, Direct control system, Angle of attack.

Chapter 4 [ORDER REDUCTION OF AIRCRAFT PITCH MODEL BY PROPOSED METHODS] Explanation of proposed model and showing its transfer function. Explanation about proposed method using in model.

Chapter 5 [RESULT] Explanation of different step response of original system and proposed method. Table of comparison analysis between routh array and pade approximation for aircraft pitch model.

Chapter 6 [CONCLUSIONS AND FUTURE SCOPE] comparative study and concludes the thesis, and proposes recommendations for future work.

Chapter 7 [LITERATURE REFERENCES] Name of References are given in this chapter.

CHAPTER 2

LITERATURE SURVEY

2.1 INTRODUCTION

Deriving reduced-order models for large-scale linear systems has been an active area of research in the control systems literature. Mathematical modeling and scientific computing are active areas of research in the control systems literature.

In the analysis and design of algorithms for complex systems, it is often necessary to derive low order models simplifying high order system models.

The use of a reduced order model makes it easier to implement analysis, simulations and control system designs.

Also, there is a great demand from the industry to use low order models rather than high order models for physical or industrial systems because it is easy to work with low order models and design controllers, observers, etc.,

Control theory can be classified as follows:
(A) Classical Control Theory
(B) Modern Control Theory

Classical control theory employs transfer functions of linear control systems, while modern control theory employs state space models for linear and nonlinear control systems. Classical and modern control methodologies are named in a misleading way, because the group of techniques called "classical" were actually developed later then the techniques labelled, “modern”. However, in terms of developing control systems, modern methods have been used to great effect more recently, while the classical methods have been gradually falling out of favour. Most recently, it has been shown that classical and modern methods can be combined to highlight their respective strengths and weaknesses.

From 1960s, there has been active research in the control literature for deriving reduced order models for large scale linear systems models, which can be classified into two broad types, viz. state-space design methods (modern control methods) and transfer function methods (classical control methods).

2.2 CLASSICAL CONTROL METHOD

The classical control methods involve the analysis and manipulation of systems in the complex frequency domain. This domain, entered into by applying the Laplace or Fourier Transforms, is useful in examining the characteristics of the system, and determining the system response.

The model reduction techniques are fundamental for the design of controllers where numerically complicated procedures are involved. The model reduction techniques enable the designer with low order controllers that may have less hardware requirements. Thus, significant efforts were taken in the control systems literature towards obtaining low-order models from high-order systems and ensuring that the reduced-order model matches some quantities of the original ones like stability, passivity, etc.

Some of the important reasons for using low-order models of high order linear systems are listed as the following:
(i) To have a better understanding of the system.
(ii) To achieve feasible controller design.
(iii) To reduce hardware complexity.
(iv) To reduce computational complexity.

Numerous methods are available in the classical control theory for the model reduction of large-scale linear control systems.

The aggregation method (Aoki, 1968) is a classical method for the model reduction of linear control systems. Moment matching technique (Sinha and Kuszta, 1983) is also a well known method for the model reduction of linear control systems.

The stability equation method (Chen, Chang and Han, 1979) is one of the popular methods for the reduction of transfer functions of high-order linear control systems. This method preserves stability in the reduced-order model if the original high-order system is stable. Some interesting variants of the stability equation method have also been obtained in the literature. Chen, Chang and Han (1980a) applied stability equation method for non-minimum phase systems. The rapos (1983) applied stability equation method to reduce the order of fast oscillating systems.

Hutton and Friedland (1975) proposed a method using Routh approximations for reducing the order of linear time-invariant systems. Appiah (1978) proposed a linear model reduction method using Hurwitz polynomial approximation. Krishnamurthy and Seshadri (1978) proposed a method of model reduction using the Routh stability criterion.

The stability equation method was also combined with Pade approximation method by Pal (1979), and Chen, Chang and Han (1980b). Wu (1981) proposed a method of model reduction using Mihailov criterion and Pade approximation technique. In order to overcome the drawback of approximating the non-dominant poles of the original system, an improved method for Pade approximants using the stable equation method was proposed by Pal (1983).

Parthasarathy and Jayasimha (1982) combined the stability equation method with modified Cauer continued fraction technique for retaining the rank of the high-order system in the reduced order model. Lamba, Gorez and Bandyopadhyay (1988) proposed a reduction technique by step error minimization for multivariable systems.

Numerous methods of obtaining reduced-order model are also available based on the minimization of the integral square error (ISE) criterion. Hwang (1984) proposed a mixed method of Routh and ISE criterion approaches for reduced order modelling of continuous time systems. Mukherjee and Mishra (1987) proposed a method of order reduction of linear systems using an error minimization technique. Puri and Lan (1988) proposed a method of obtaining stable model reduction by impulsive response error minimization using Mihailov criterion and Pade approximation. Vilbe and Calvez (1990) proposed a method of order reduction of linear systems using an error minimization technique. Mittal, Prasad and Sharma (2004) also proposed a method of order reduction of linear dynamic systems using an error minimization technique. A common feature in these model reduction methods is that the values of the denominator coefficients of the low order system are chosen arbitrarily by some stability preserving methods such as dominant pole, Routh approximation methods, etc. and the numerator coefficients of the low order system are determined by the minimization of the integral square error (ISE).

Hewitt and Luss (1990) proposed a model reduction technique in which both the numerator and denominator coefficients are considered to be free parameters and chosen to minimize the ISE in impulse or step responses. Safonov, Chiang and Limebeer (1990) proposed optimalHankel model reduction for non-minimal systems. Prasad (2000) proposed a Pade type model reduction for multivariable systems using Routh approximation.

Numerous methods of classical control theory have been also proposed for the model reduction of linear discrete-time control systems. Shamash and Feinmesser (1978) proposed a method of model reduction of discrete-time systems using Routh array. Bistritz and Shaked (1984) proposed a method of approximating discrete multivariable system by minimal Pade type stable model. Bandyopadhyay and Kande (1988) proposed a method of model reduction for discrete-time control systems by Pade type approximations.

Hsieh and Hwang (1990) proposed a method of model reduction of linear discrete-time systems using bilinear Schwarz approximation. Prasad (1993) proposed a method of order reduction of discrete time systems using stability equation method and weighted time moments. Madievski and Anderson (1995) proposed a procedure for sampled-data controllers. Wang and Zilouchian (2000) proposed a method of model reduction of discrete linear systems via frequency-domain balanced structure.

Mukherjee, Kumar and Mitra (2004) proposed a method of order reduction of linear discrete system using an error minimization technique. Recently, Saras wathi (2011) has proposed a modified method for order reduction of large scale discrete systems. In this paper, a new procedure has been proposed for evaluating time moments of the original high order system, and numerator and denominator polynomials of reduced order model are obtained by considering first few redefined time moments of the original high order system.

Evolutionary algorithms have been also used for the order reduction of large-scale linear control systems. Particle Swarm Optimization (PSO) is a population based stochastic optimization technique (Kennedy and Eberhart, 1995), which is inspired by social behaviour of bird flocking. PSO shares many common features with evolutionary computing techniques such as Genetic Algorithms (GA).

Parmer and Mukherjee (2007) proposed a method for reduced order modelling of lineardynamic systems using particle swarm optimized eigen spectrum analysis. Recently, Saini and Prasad (2010) proposed a method for the reduction of linear interval systems using genetic algorithms. Kumar, Ghosh and Mukherjee (2011) have proposed a method of model order reduction using bio-inspired Particle Swarm Optimization (PSO) and Bacterial Foraging Optimization (BFO) soft-computing for comparative study. In this paper, the numerator and denominator polynomial of the reduced order model of high order linear dynamic systems are computed by minimizing the integral square error between the original high order and reduced order system using PSO and BFO.

2.3 MODERN CONTROL METHODS

The modern control methods use state space models for the design methodology. The state variable model is broad enough to be useful in describing a wide range of systems, including systems that cannot be adequately described using the Laplace Transform.

Physical systems are often modeled in the so-called "time domain", where the response of a given system is a function of the various inputs, the previous system values, and time. Modern control methods are carried out in the state space model of a system. Thus, the modern control methods can easily deal with multi-input and multi-output (MIMO) systems. Thus, the modern control theory overcomes the limitations of classical control theory in more sophisticated design problems such as fighter aircraft control and satellite control problem which have the limitation that no frequency domain analysis is possible.

Numerous papers have been published using modern control theory for the model reduction of linear control systems. Santiago and Jamshidi (1986) proposed a balanced approach for the model reduction of linear control systems.

Aldeen (1991) derived results for interaction modelling approach to distributed control with application to interconnected dynamical systems. Aldeen and Trinh (1994) derived results for observing a subset of the states of linear control systems. Grimme (1997), Kamon, Wang and White (2000) and Salimbahrami and Lohmann (2006) obtained results for the model reduction of linear control systems using Krylov subspace methods.

Benner and Quintana-Orti (2003) proposed state-space truncation methods for parallel model reduction of large scale linear systems. Nagar and Singh (2004) developed an algorithmic approach for system decomposition and balanced model reduction for linear systems. Shieh and Wei (1975) proposed a mixed method for multivariable linear system reduction.

Condon, Ivanov and Brennan (2005) obtained a causal model for linear RF systems developed from the frequency domain measured data. Willcox and Megretski (2005) applied Fourier series for accurate stable reduced-order models in large-scale linear applications.

Sundarapandian (2005) developed distributed control schemes for large scale interconnected discrete-time linear systems.

Vishwakarma and Prasad (2008) used clustering method for deriving reduced order models of linear dynamic systems. Reduced order models were derived for interval systems by Sastry, Raja Rao and Rao (2000), Dolgin and Zeheb (2003), Saini and Prasad (2010), and Kumar, Nagar and Tiwari (2011).

David Luenberger's seminal paper titled "Observers for multivariable systems" (1966) presents three different types of observers, viz.

(1) The Full-State Observer

(2) Reduced-Order Observer

(3) Functional Observer

There has been significant interest in the literature on the third type of observer, viz. the functional observer, which reconstructs a single linear functional of the unknown state vector of the form $\xi = Kx$. It is well-known that only a linear function of the system state vector is required for a stabilizing or regulating feedback and for this situation, the design of a functional observer that estimates ξ having lower dimension than the state x will be useful.

Fortmann and Williamson (1972) derived necessary and sufficient conditions for the existence of a functional observer for linear continuous-time control systems. Murdoch (1974) derived results for degenerate observers for linear continuous-time control systems. Moore and Ledwich (1975) used decision theory to derive necessary and sufficient conditions for the design and existence of a p^{th} order observer.

Gupta, Fairman and Hinamoto (1981) proposed a method for the design of an observer for the reconstruction of a single linear functional of the state variables for a linear continuous-time system. Tsui (1985) presented some modifications to the work of Gupta *et.al.* (1981) and the most significant improvement was the use of a transformation to place the state matrix Ain lower Hessenberg form.

Aldeen and Trinh (1999) proposed a design method for the functional observer, which bases the observer order on the ratio of independent output measurements to independent input measurements. This method is very useful for high order systems having far more number of outputs than inputs.

2.4 MODEL ORDER REDUCTION METHODS

Reduced order modeling has wide applications in various fields of engineering and therefore order reduction methods have been discussed in details in literature during last few decades and in text books. The model order reduction techniques are broadly classified in frequency domain and time domain reduction methods.

2.5 FREQUENCY DOMAIN ORDER REDUCTION METHODS

The frequency domain order reduction techniques can be subdivided into three groups

(i) Classical Reduction Methods (CRM)

(ii) Stability Preservation Methods (SPM)

(iii) Stability Criterion Methods (SCM)

The methods of first group are based on classical theories of mathematical approximation such as continued fraction expansion and truncation, pade approximation, time moment matching etc. These methods are algebraic in nature and some cases may reduce unstable system to stable system and vice versa. The problems such as instability, non minimum phase behavior and accuracy in the mid and high frequency range of reduced order models limit the application of Classical Reduction Method (CRM).

Stability Preservation Methods (SPM) is stable reduction methods. These methods suffer from a serious drawback of flexibility when the reduced order model does not produce a good enough approximation. This group includes Routh approximations, Hurwitz polynomial approximation, Routh – Hurwitz array and stability equation method. The other SPM are dominant pole retention, reduction based on differentiation, the method using Mihailov Criterion also preserve stability and can be included in SPM.

Stability Criterion Methods (SCM) includes mixed methods. In these methods, the denominator of reduced order model is derived by one of stability preservation methods

(SPM), while numerator coefficients of the reduced order model are determinedby using one of the classical reduction methods (CRM), to improve the degree of accuracy at low frequency range.

Some important frequency domain order reduction methods are briefly reviewed below:

2.5.1 Continued Fraction Expansion (CFE) Method:

Chen and Shieh proposed this method of obtaining reduced order model of linear time invariant SISO system. A detailed account of continued fraction expansion is available in Wall.

This approach does not require any knowledge of Eigen values or Eigen vectors and contains most of the essential characteristics of original system in first few terms. The basic principle gives rise to the derivation of simplified models by continued fraction expansion (CFE) is based on expanding the original higher order system using continued fraction expansions. As the quotients in continued fraction expansion descend lower and lower in position, they become less and less important as far as their influence on the performance of the system is concerned. This observation is general basis of the simplification techniques using continued fraction expansions. After truncating, the continued fraction expansion after some terms, and inversion the truncated CFE, results in a reduced order model. The various modifications and extensions have been carried out by many authors. Davidson and Lucas give CFE about a general point; Chen extended CFE to MIMO systems.

2.5.2 Moment Matching Method:

This model order reduction approach of moment matching was first introduced by Paynter and Zarkian applied this method. This method is based on determining a set of time functions of full order model and matching them with those for the reduced order model i.e. matching a few lower order moments of original high order system with that of the reduced order model. Matching of initial time-moments leads to better approximation at low frequency and matching of initial Markov parameters leads to better approximation at high frequencies.

This method is the transient performance of the reduced order model may not always be satisfactory and also there is no guarantee of stability. This method has calculation difficulty for large no of large time constant.

2.5.3 Pade Approximation Method:

Pade introduced this technique and Shamash applied this method. This method is computationally simple, fits initial time moments and steady state value of original and reduced order model matches. For rth order model, '2r-1' coefficients of power series expansion (about s=0) of reduced order model matched with the corresponding coefficients of the original system.

The disadvantage this method is that reduced order model may be unstable even though the original system is stable. Also it may sometimes approximate non –dominant poles of the system, thus giving bad approximation. To overcome this disadvantage, various alternatives methods have been suggested. Shamash introduced a method of reduction based on retention of poles of high order system in reduced order model and concept of Pade approximation about more than one point.

2.5.4 Routh Approximation Method:

This method of model order reduction was proposed by Hutton and Friend land. The transfer function of high order system is initially reciprocated and then expanded in the □-β canonical form for the denominator and numerator polynomials respectively. The □ table is prepared from denominator coefficients using well known Routh algorithm where β table is prepared by similar algorithm using numerator coefficients in which β coefficients are determined by using the □-table and successive elements of β- table.

This method requires neither optimization nor Eigen values evaluation, but ensures system stability and the steady state values of reduced order models match with that of original system. It involves simple algebraic calculations of finite number of steps.

2.5.5 Routh –Hurwitz Array Method:

This technique consists of obtaining the numerator and denominator polynomials of the reduced order model respectively from the numerator and denominator polynomials of the original system by forming the Routh Hurwitz stability arrays for numerator and denominator polynomials. Using second and third rows of Routh stability array nth degree denominator polynomial, a polynomial of (n-1)th order can be constructed. Similarly (n-2)th order polynomial can be constructed from third and fourth row of the array and so on. The same procedure is repeated for reducing the numerator polynomial.

2.5.6 Stability Equation Method:

This method was proposed by Chen et al. In this method, numerator and denominator polynomials of high order system are separated into their even and odd parts. These are then factored and roots, which are closer to origin, are retained.

In this method, polynomial is reduced by successively discarding the less significant factors. The transfer function of the reduced order model is constructed using these retained roots. This method preserves stability in the reduced order model for stable original system and retains the first two time moments of the system, thus ensuring steady state response matching for impulse, step and ramp inputs.

2.5.7 Polynomial Differentiation Method:

This method introduces by Gutman et al. The reciprocal of numerator and denominator polynomials of high order transfer function are differentiated many times to yield the coefficients of reduced order transfer function. These reduced polynomials are reciprocated back and normalized. This method is computationally simple and is applicable to unstable and non-minimum phase systems. The only drawback of the method is that steady state does not match always.

2.5.8 Truncation Method:

It was first introduced by Gustafson where successively lower order models are obtained by neglecting progressively higher order terms from numerator and denominator of high order system. This method is computationally simple. Shamash extended this method for multivariable systems and compares the technique of Routh approximation and concluded this method is equally good as other methods.

2.5.9 Dominant Pole Retention Method:

This approach was proposed by Davison. In this method the reduced order model is always stable for a stable original system and dominant performance of the original system is also retained. The poles near to imaginary axis, known as dominant poles are retained in reduced order model and poles, far away from imaginary axis are neglected, as their effect on overall performance of the system is comparatively less. The disadvantage of this method is that if all the poles are very near to imaginary axis then it becomes difficult to distinguish which one is more dominant.

2.5.10 Factor Division Method:

Lucas introduced this technique for model order reduction. It allows retention of dominant poles and initial time moments in the reduced order model. It avoids calculation of system time moments and the solution of Pade equation by dividing out the unwanted poles factors. This method was extended by Lucas to generate biased order models by retaining initial Markov parameters as well as time moments. The ideas of Lucas were extended and modified factor division approach was developed.

2.5.11 Mihailov Stability Criterion:

Mihailov stability criterion of model order reduction was described by D. Kranthi at el.D.Kranthi combined this criterion with the Factor division approach. This method is computationally simple and efficient. It avoids calculating the initial time moments Markov parameters of the original system. This methods also ensures stability if original system is stable.

2.5.12 Least Square Method:

Shoji et al. introduced the method of model order reduction based on least square matching of time moments of the original system. An attractive feature of this method is that it provides an extra degree of freedom in the design of reduced order model. Lucas and Beat modified this method. Smith and Lucas estimated the denominator coefficients by least square method and numerator coefficients by exact moment matching.

2.6 TIME DOMAIN ORDER REDUCTION METHODS

The time domain order reduction methods require either the major knowledge of overall characteristics or the Eigen values and Eigen vectors of higher order systems. Some important time domain order reduction methods are also briefly reviewed below:

2.6.1 Aggregation Method:

In this method which retains the important Eigen values of the original system in the reduced model, the most general is the aggregation proposed by Aoki. Aoki has shown usefulness of aggregation matrices for designing suboptimal controllers.

2.6.2 Singular Perturbation Method:

The singular perturbation method introduced by Sannuti and Kokotovic. It is particularly useful for simplifying a system having the time scale property. Then states are portioned into the slow and fast parts, and reduction is obtained by setting the derivatives of fast states to zero, so that the fast states can be eliminated. An important advantage of the method is that the physical nature of the problem is persevered. The main difficulty with this method is determining an appropriate partitioning of the state vector to obtain a suitable low order model.

2.6.3 Optimal Order Reduction Methods:

This group of methods is based on obtaining a model of specified order such that its impulse or step response (or alternately its frequency response) matches that of the original system in optimum manners with no restriction on the location of the Eigen values. Such techniques aim minimizing a selected performance criterion which in general is a function of error between the response of the original high order system and its reduced order model.

The parameter of reduced order model are then obtained either from the necessary conditions of optimality or by means of numerical algorithm. Anderson [45] proposed a geometric approach, based on orthogonal projection, to obtain a low order minimizing the integral square error in time domain. Sinha and Pille proposed utilizing the matrix pseudo inverse for a least squares fit with samples of the response.

Other criteria for optimization have been studied (Sinha and Bereznai, and suggested using the pattern search method of Hooke and Jeeves, where as Bandler, Markettos and Sinha have proposed using gradient methods, which require less computation time but the gradient of objective function has to be evaluated. The development of optimal order reduction is attributed to Wilson and Mishra. Where the approximation has been studied for step and impulse responses.

2.7 CONCLUSION

The various model order reduction methods proposed and applied in the design and development by various researchers have been described. These methods are segmented into frequency domain and time domain reduction methods. Depending on the ways of methods is further classified as Classical reduction methods, stability preservation methods and stability criterion methods in frequency domain. The methods which are used and exploited further in the coming chapters have been described.

CHAPTER 3

AIRCRAFT PITCH MODEL

3.1 AIRCRAFT PRINCIPAL AXES

An aircraft in flight is free to rotate in three dimensions: *yaw*, nose left or right about an axis running up and down; *pitch*, nose up or down about an axis running from wing to wing; and *roll*, rotation about an axis running from nose to tail. The axes are alternatively designated as *vertical*, *transvers*and *longitudinal* respectively. These axes move with the vehicle and rotate relative to the Earth along with the craft. These definitions were analogously applied to spacecraft when the first manned spacecraft were designed in the late 1950s.

These rotations are produced by torques (or moments) about the principal axes. On an aircraft, these are intentionally produced by means of moving control surfaces, which vary the distribution of the net aerodynamic force about the vehicle's canter of gravity. Elevators (moving flaps on the horizontal tail) produce pitch, a rudder on the vertical tail produces yaw, and ailerons (flaps on the wings that move in opposing directions) produce roll. On a spacecraft, the moments are usually produced by a reaction control system consisting of small rocket thrusters used to apply asymmetrical thrust on the vehicle.

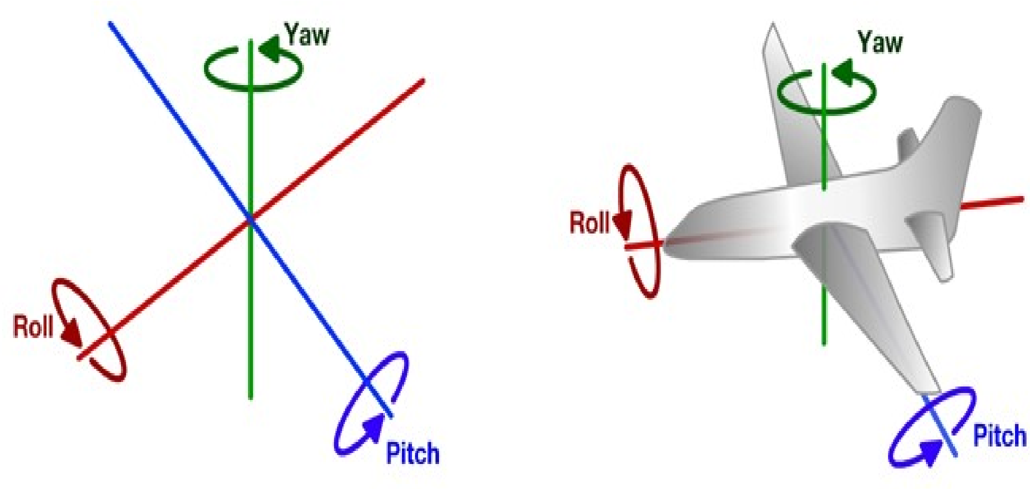

Figure 3.1 Aircraft Principle axes

3.2 PRINCIPAL AXES

There are three main axes of aircraft:

1. Normal axis, vertical axis or yaw axis — an axis drawn from top to bottom and perpendicular to the other two axes. Parallel to the *fuselage station*.
2. Transverse axis, lateral axis, or pitch axis — an axis running from the pilot's left to right in piloted aircraft, and parallel to the wings of a winged aircraft. Parallel to the *buttock line*.
3. Longitudinal axis or roll axis — an axis drawn through the body of the vehicle from tail to nose in the normal direction of flight, or the direction the pilot faces. Parallel to the *waterline*.

Normally, these axes are represented by the letters X, Y and Z in order to compare them with some reference frame, usually named x, y, z. Normally, this is made in such a way that the X is used for the longitudinal axis, but there are other possibilities to do it.

3.2.1 Vertical axis (yaw)

The **yaw axis** has its origin at the center of gravity and is directed towards the bottom of the aircraft, perpendicular to the wings and to the fuselage referenceline. Motion about this axis is called **yaw**. A positive yawing motion moves the nose of the aircraft to the right. The rudder is the primary control of yaw.

The term *yaw* was originally applied in sailing, and referred to the motion of an unsteady ship rotating about its vertical axis. Its etymology is uncertain.

3.2.2 Transverse axis (pitch)

The **pitch axis** (also called **transverse** or **lateral axis**) has its origin at the center of gravity and is directed to the right, parallel to a line drawn from wingtip to wingtip. Motion about this axis is called **pitch**. A positive pitching motion raises the nose of the aircraft and lowers the tail. The elevator is the primary control of pitch.

3.2.3 Longitudinal axis (roll)

The **roll axis** (or **longitudinal axis**) has its origin at the center of gravity and is directed forward, parallel to the fuselage reference line. Motion about this axis is called **roll**. An angular displacement about this axis is called **bank**. A positive rolling motion lifts the left wing and lowers the right wing. The pilot rolls by increasing the lift on one wing and decreasing it on the other. This changes the bank angle. The ailerons are the primary control of bank. The rudder also has a secondary effect on bank.

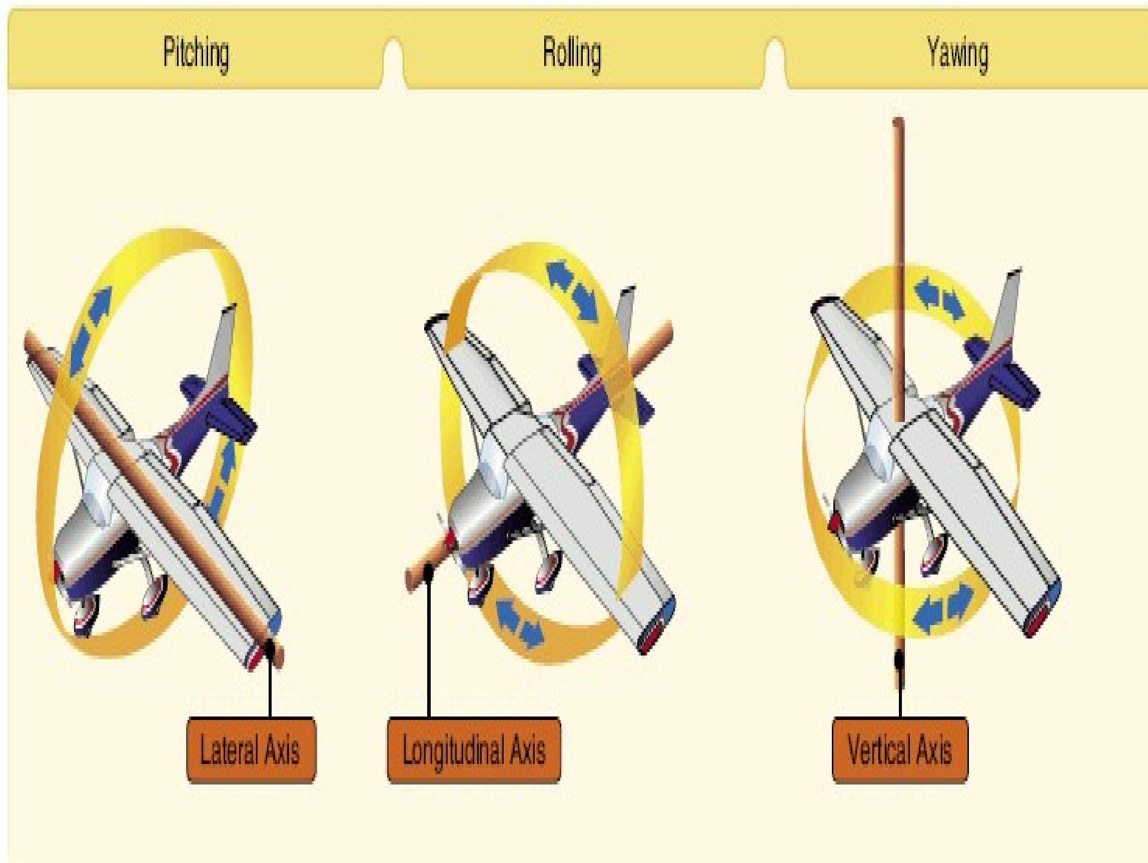

Figure 3.2 Lateral, Longitudinal, Vertical Axes

3.3 The Gain Changer

Imagine being the pilot of a typical small airplane, perhaps a Cessna 172. For the stable flight regime in which the plane is operated, the control stick has a ‘feel’ that the pilot has come to expect. A given user input to the control stick results in a known result that isn’t touchy or sluggish. In the longitudinal axis, when the control stick is deflected, the aircraft responds by rotating nose up or nose down about the center of gravity. The rate of rotation is called the pitch rate.

Unfortunately, as the center of gravity of the airplane changes, intentionally or not, the relationship between the control stick and the resulting output changes. Figure 0.1 shows the relationship between User Input and Pitch Rate Output as a scalar, called gain ‘A’. ‘A’ represents DC gain, and is a measured in a steady state condition.

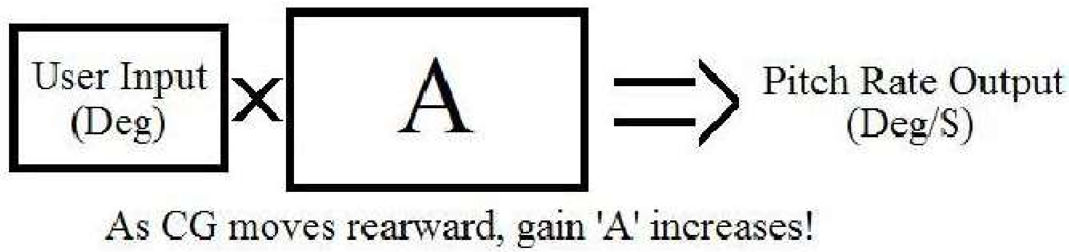

Figure 3.3 User Input and Pitch Rate Output

For a given User Input, as the CG moves rearward, gain ‘A’ increases, resulting in a larger pitch rate. At some point the gain ‘A’ is large enough that control stick becomes so sensitive that the aircraft is difficult, if not impossible to fly. This change in the gain ‘A’ is the pitch rate gain variance with respect to CG. Minimizing the gain variance with respect to CG movement is the focus of this work.

3.4 PITCH CONTROL AND STABILITY THEOTY

3.4.1 Pitch Control:

The elements of a typical aircraft control system are shown in Figure 3.4.1

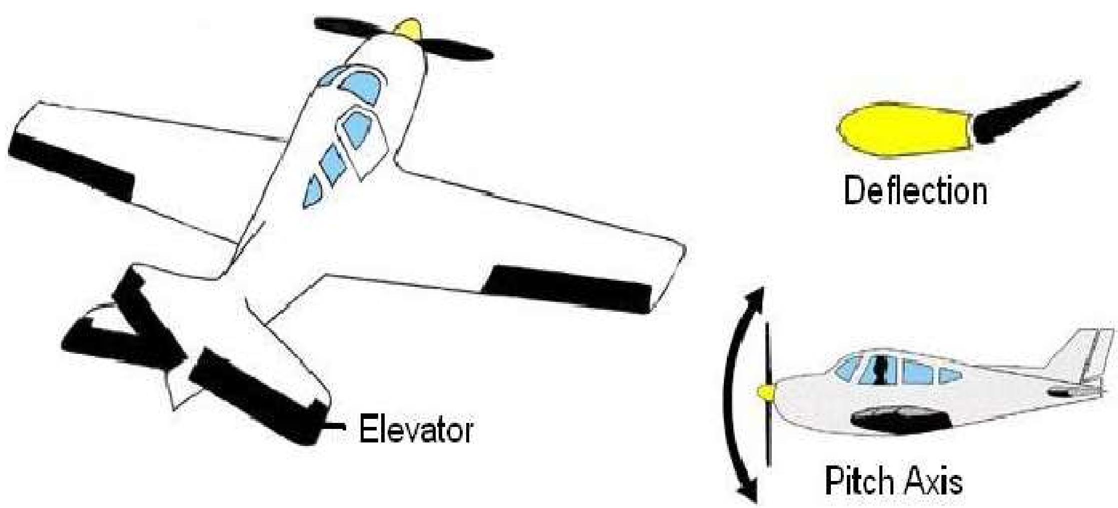

Figure 3.4.1 Elevators and the Pitch Axis

The control of an airplane's pitch is dependent on the deflection of its elevator, a hinged surface located at the tail of the airplane shown above. Airflow is redirected when the elevator is displaced. This causes a force and as a result the aircraft revolves about its pitch axis, located at the longitudinal center of gravity. In order to displace the elevator a control stick located in the cockpit is rotated forward or aft. The elevator deflects in proportion to the degrees of stick rotation (Deg), and the resulting rotation about the CG is called the pitch rate, measured in degrees of rotation per second (Deg/S).

3.4.2 Aircraft Stability

Consider Figure 1.2. Note that the center of lift (CL) is a point that exists ¼ of the way backfrom the leading edge of the wing. The center of gravity is point which moves based on theweight distribution of an aircraft. Traditionally an aircraft is expected to be stable if thecenter of gravity (CG) is located in front of the center of lift (CL). If the center of gravitywas located at the same place as the center of lift, the airplane would be neutrally stable. Ifthe CG was moved behind the center of lift, the aircraft would be unstable.

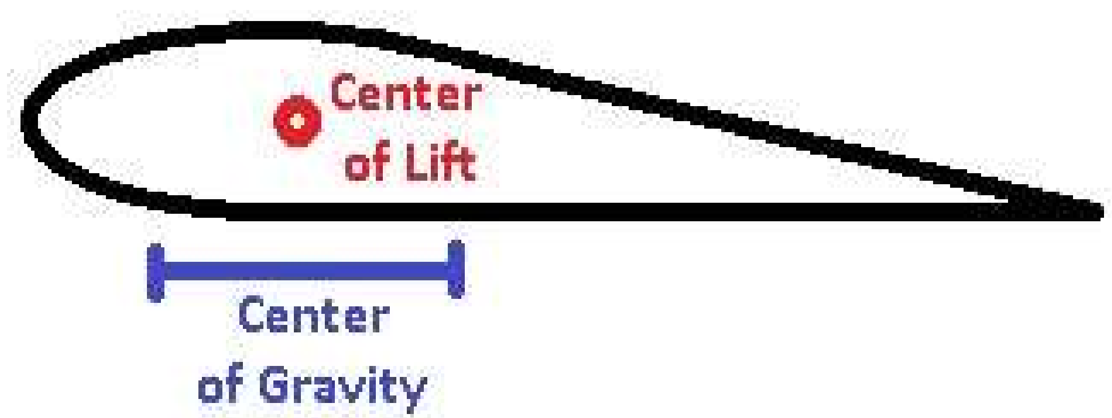

Figure 3.4.2 Airfoil, Center of Lift, and Center of Gravity Range

For the purposes of this work, it is important to understand two types of stability.

Static stability refers to the initial response of an aircraft after a perturbation from equilibrium. An aircraft that tends to return to equilibrium after displacement exhibits a restorative force called subsidence, and is statically stable. If the aircraft tends to depart further from the equilibrium point after a disturbance, it exhibits divergence and is statically unstable. If a disturbance does not result in the generation of either a restoring or departing force, the aircraft is *neutrally stable*; this condition represents the boundary between stable and unstable.

Dynamic stability is represented by the time history of motion of an aircraft after it has been disturbed or a user input commands it from equilibrium. If an aircraft at equilibrium was being displaced, the reduction of disturbance with time would imply a resistance to motion; An aircraft with negative damping that was displaced from equilibrium would continue to diverge from equilibrium. This departure could take the form of an exponential divergence or growing oscillation. Any negatively damped aircraft would require constant pilot attention and continuous correction, if it was flyable at all.

In cases of negatively damped aircraft, a closed loop control system can be employed to provide restorative forces. This generally consists of an electromechanical system which senses undesirable motion and responds by damping that motion.

3.5 DESCRIPTION OF AIRCRAFT FLIGHT CONTROL

The four basic forces acting upon an aircraft during flight are lift, weight, drag and thrust as shown in Figure 3.5

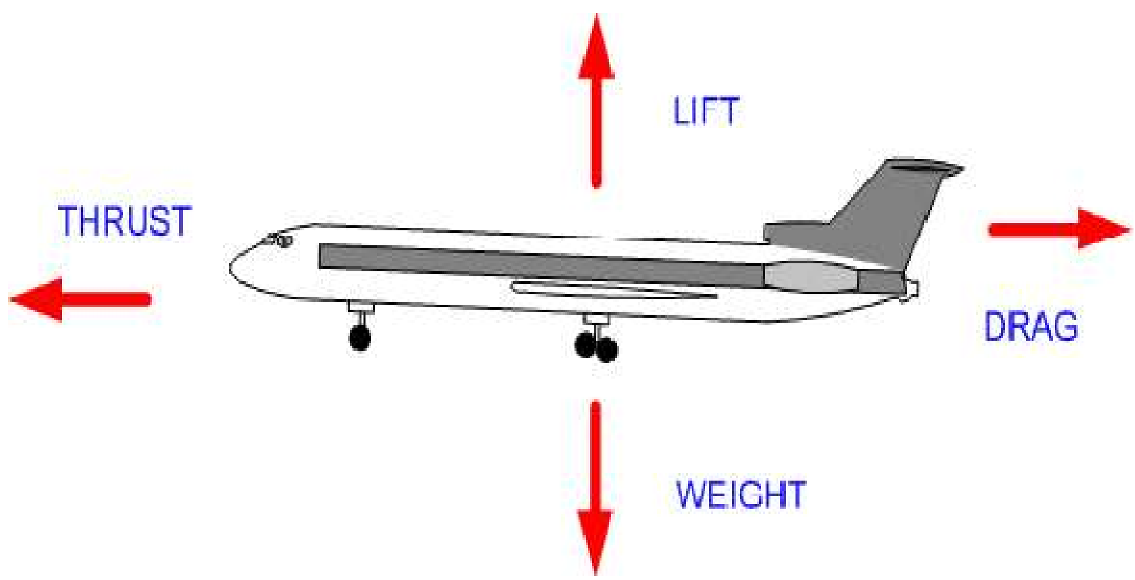

Figure 3.5 Forces acting on an aircraft

3.5.1 Lift

Due to flow around the aircraft, it causes the Lift. Upward force created by wings is the Lift, which sustains the aircraft in flight. The force required to lift the plane through a stream of air depends upon the wing profile. When the lift is greater than the weight then the plane raises.

3.5.2 Weight

Due to the weight of plane, downward force is created on the aircraft and it is directly proportional to lift. If the weight is more than lift then the plane descends.

3.5.3 Drag

The drag of the aircraft to forward motion directly opposed to thrust. The resistance of the air makes it hard for the aircraft to move fast. Another name for drag is air resistance.

3.5.4 Thrust

The aircraft in forward direction due to force exerted by the engine which pushes air backward with body of aircraft causing a reaction or thrust.

3.6 AIRCRAFT FLIGHT CONTROL SURFACES

To move in different directions an aircraft need control surface to fly. Control surface makes it possible for the aircraft to roll, pitch and yaw. Figure 1.2 shows the three sets of control surfaces and the axes along the aircraft.

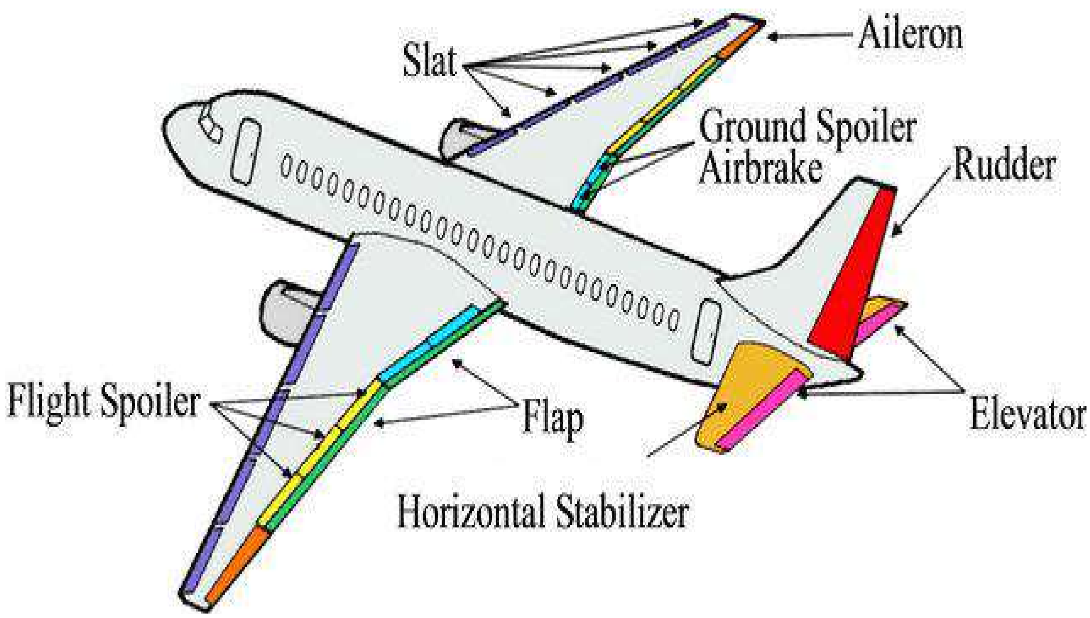

Figure 3.6 (a) Aircraft Flight Control Surface

The ailerons, operated by turning the control column (Figure 3.6 b), cause it to roll. The elevators are operated by moving the control column forward or back causes the aircraft to pitch. The rudder is operated by rudder pedals that make the aircraft yaw. Depending on the kind of aircraft, the requirements for flight control surfaces vary greatly, as specific roles, ranges and needed agilities. Primary control surfaces are incorporated into the wings and empennage for almost every kind of aircraft. Those surfaces are typically: the elevators included on the horizontal tail to control pitch; the rudder on the vertical tail for yaw control; and the ailerons outboard on the wings to control roll.

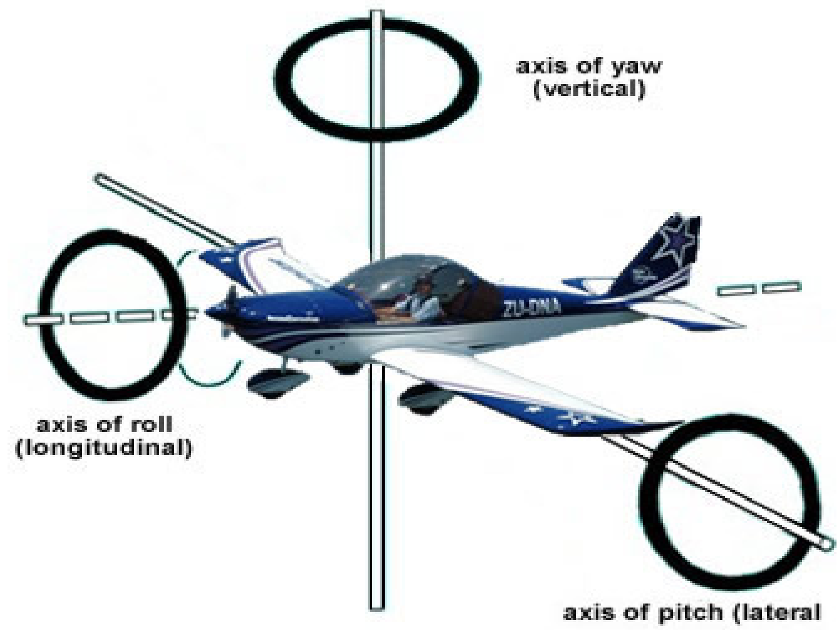

Figure3.6(b) Axes of Aircraft

These surfaces are continuously checked to maintain safe vehicle control and they are normally trailing edge types.

3.7 AIRCRAFT PRIMARY CONTROL SURFACE

There are three types of primary control surfaces as ailerons, elevator and rudder.

3.7.1 Ailerons

When the aircraft move about the longitudinal axis is controlled by the two ailerons, which are movable surfaces at the outer trailing edge of each wing. The movement is roll. If the aileron on one wing is down, the aileron on the other will be up. The wing with the up aileron goes down because of its loose lift and the wing with the lowered aileron goes up because of its increased lift.

The ailerons are connected to each other and to the control wheel (or stick) in the cockpit by rods or cables. While applying pressure to the right on the control wheel, the right aileron goes up and the left aileron goes down. Thus, the airplane is rolled to the right as the down movement of the left aileron increases the wing camber (curvature) and 10 theangle of attack. The right aileron moves upward and decreases the camber, what results in a decreased angle of attack. Thus, an increased lift on the left wing and decreased lift on the right wing cause a roll and bank to the right.

3.7.2 Elevators

The elevators are control the movement of aircraft about the lateral axis. This motion is called pitch. The elevators are free to swing up and down and form the rear part of the horizontal tail assembly. They are hinged to a fixed surface, the horizontal stabilizer. A single airfoil is formed by the horizontal stabilizer and the elevators, which increases or decreases the lift.

In the aircraft control cables are used to connect the elevators to the control wheel as it happens with the ailerons. The elevators move downward when forward pressure is applied on the wheel. Thus, the lift produced by the horizontal tail surfaces is increased, what forces the tail upward, causing the nose to drop. Similarly, the elevators move upward, when back pressure is applied on the wheel, decreasing the lift produced by the horizontal tail surfaces, or maybe even producing a downward force. The nose is forced upward and the tail is forced down.

In the aircraft the angle of attack of the wings is controlled by the elevators. When back pressure is applied on the control wheel, the angle of attack increases as the tail lowers and the nose rises. Similarly, the tail raises and the nose lowers when forward pressure is applied, decreasing the angle of attack.

3.7.3 Rudder

The rudder is the control of movement aircraft about the vertical axis. This motion is called yaw. The rudder is a movable surface hinged to a fixed surface which is the vertical stabilizer, or fin. Its action is similar to the one of the elevators, except that it swings in a different plane; from side to side instead of up and down. The rudder is connected to the rudder pedals by controlled cables.

3.8 AIRCRAFT SECONDARY CONTROL SURFACES

During take-off and landing of an aircraft wing Leading and Trailing edges are used to increase the aerodynamic performance of the aircraft by reducing stall speed. High lift control is provided by a combination of flaps and leading edge slats. The flap control is affected by several flap sections located on the inboard two-thirds of the wing trailing edges. The flaps are deployed during take-off or the landing approach to increase the wing area and improve the aerodynamic characteristics of the wing.

3.8.1 Flaps

The location of flaps are mounted on the trailing edge but can also be mounted on the leading edge. They extend the edge by increasing the chord of the wing. They pivot only, extend and come down or extend and area.

3.8.2 Slats

The location of slats is usually mounted on the leading edge. Slats extend the edge and they sit like a glove on the edge. Slats means they have a nozzle like slot between the high-lift device and the wing, on the contrary, flaps do not have this slot. Figure 4.7.2 shows the wing leading and trailing edge configurations commonly used.

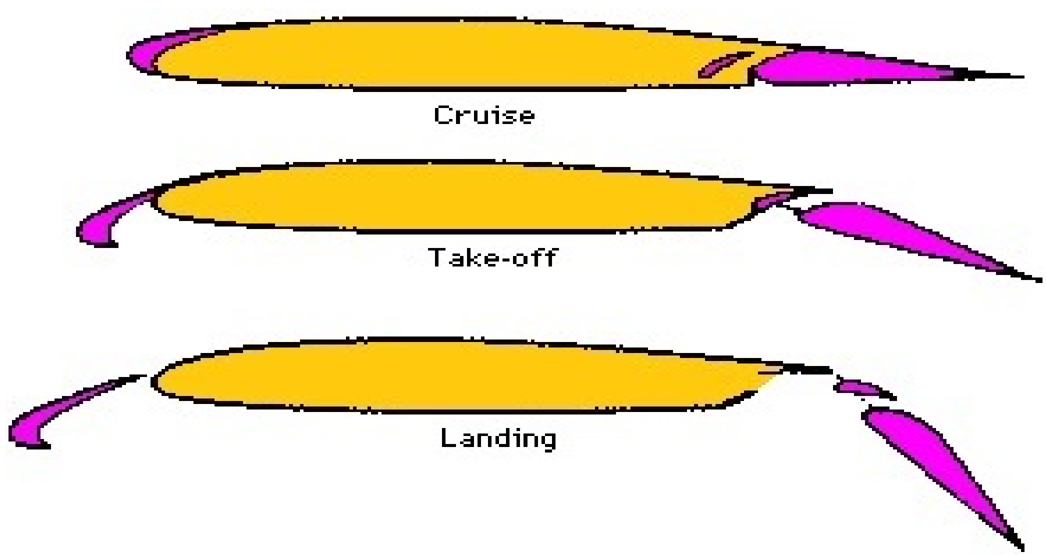

Figure 3.8.2 Slats of Aircraft

3.9 DIRECT CONTROL SYSTEM

The linkage from cabin to control surface can be fully mechanical if the aircraft size and its flight envelop allow; in this case the hinge moment generated by the surface deflection is low enough to be easily contrasted by the muscular effort of the pilot. Two types of mechanical systems are used: push-pull rods and cable-pulley.

In the first case a sequence of rods link the control surface to the cabin input. Bell-crank levers are used to change the direction of the rod routings: Fig 3.9 sketches the push-pull control rod system between the elevator and the cabin control column; the bell-crank lever is here necessary to alter the direction of the transmission and to obtain the conventional coupling between stick movement and elevator deflection (column fwd = down deflection of surface and pitch down control). First of all the linkage must be stiff, to avoid any unwanted deflection during flight and due to fuselage elasticity. Second, axial instability during compression must be excluded; the instability load P for a rod is given by:

$$P= \pi^2 E I / \lambda 2 \qquad \ldots\ldots (3)$$

Where:

E = Young modulus;

I = cross-section moment of inertia;

λ = reference length.

The reference length is linked to the real length of the rod, meaning that to increase the instability load the length must be decreased, or the rods must be frequently constrained by slide guides, or the routing must be interrupted with bell-cranks. Finally a modal analysis of the system layout is sometimes necessary, because vibrations of the rods can introduce oscillating deflections of the surface; this problem is particularly important on helicopters, because vibrations generated by the main rotor can induce a dramatic resonance of the flight control rods. The same operation described before can be done by a cable-pulley system, where couples of cables are used in place of the rods. In this case pulleys are used to alter the direction of the lines, equipped with idlers to reduce any slack due to structure elasticity, cable strands relaxation or thermal expansion.

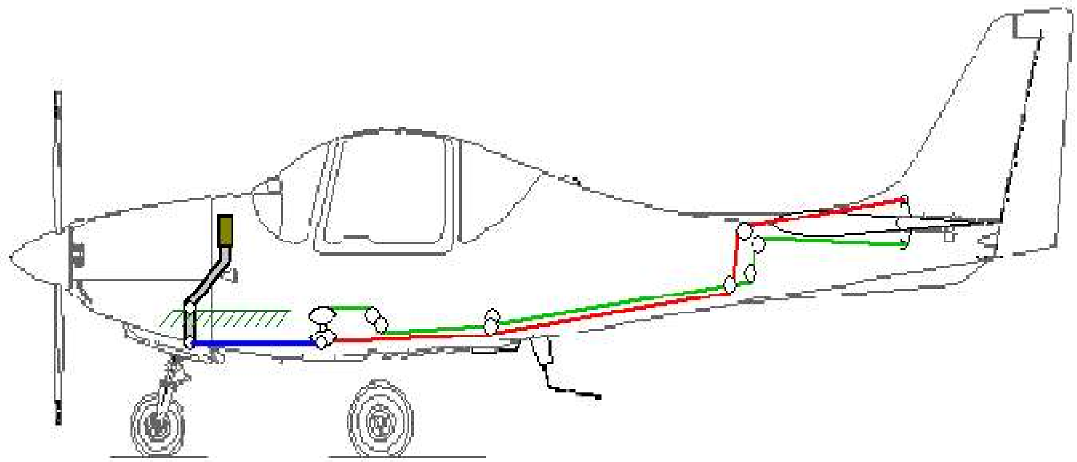

Figure 3.9 Cables and pulleys system for elevator control

3.10 Fly-By-Wire

The cable-pulley solution is often preferred, because is more flexible and allows reaching more remote areas of the airplane. An example, where the cabin column is linked via a rod to a quadrant, which the cables are connected too.

In the 70"s the fly-by-wire architecture was developed, starting as an analogue technique and later on, in most cases, transformed into digital. It was first developed for military aviation, where it is now a common solution; the supersonic Concorde can be considered a first and isolated civil aircraft equipped with a (analogue) fly-by-wire system, but in the 80"s the digital technique was imported from military into civil aviation by Airbus, first with the A320, then followed by A319, A321, A330, A340, Boeing 777 and A380. This architecture is based on computer signal processing and the pilot's demand is first of all transducer into electrical signal in the cabin and sent to a group of independent computers (Airbus architecture substitute the cabin control column with a side stick); the computers sample also data concerning the flight conditions and servo-valves and actuators positions; the pilot's demand is then processed and sent to the actuator, properly tailored to the actual flight status.

The flight data used by the system mainly depend on the aircraft category; in general the following data are sampled and processed:

1. Pitch, roll, yaw rate and linear accelerations
2. Angle of attack and side slip
3. Airspeed/Mach number, pressure altitude and radio altimeter indications

4. Stick and pedal demands
5. Other cabin commands such as landing gear condition, thrust lever position, etc.

The full system has high redundancy to restore the level of reliability of a mechanical or hydraulic system, in the form of multiple (triplex or quadruplex) parallel and independent lanes to generate and transmit the signals, and independent computers that process them; in many cases both hardware and software are different, to make the generation of a common error extremely remote, increase fault tolerance and isolation; in some cases the multiplexing of the digital computing and signal transmission is supported with an analogue or mechanical back-up system, to achieve adequate system reliability.

The benefits of the fly-by-wire architecture are different, and vary significantly between military and civil aircraft; some of the most important benefits are as follows:

i. Flight envelope protection (the computers will reject and tune pilot's demands that might exceed the airframe load factors)
ii. Increase of stability and handling qualities across the full flight envelope, including the possibility of flying unstable vehicles
iii. Turbulence suppression and consequent decrease of fatigue loads and increase of passenger comfort
iv. Use of thrust vectoring to augment or replace lift aerodynamic control, then extending the aircraft flight envelope
v. Drag reduction by an optimized trim setting
vi. Higher stability during release of tanks and weapons
vii. Easier interfacing to auto-pilot and other automatic flight control systems
viii. Weight reduction (mechanical linkages are substituted by wirings)
ix. Maintenance reduction
x. Reduction of airlines‟ pilot training costs (flight handling becomes very similar in an whole aircraft family).

For civil fly-by-wire aircraft in normal operation the flight control changes according to the flight mode: ground, take-off, flight and flare. Transition between modes is smooth and the pilot is not affected in its ability to control the aircraft: in ground mode the pilot has control on the nose wheel steering as a function of speed, after lift-off the envelope protection is gradually introduced and in flight mode the aircraft is fully protected by exceeding the maximum negative and positive load factors (with and without high lift devices extracted), angle of attack, stall, airspeed/Mach number, pitch attitude, roll rate, bank angle etc.; finally, when the aircraft approaches to ground the control is gradually switched to flare mode, where automatic trim is deactivated and modified flight laws are used for pitch control.

3.11Aircraft Actuation System

Actuation systems are a vital link in the flight control system, providing the motive force necessary to move flight control surfaces. Whether it is a primary flight control, such as an elevator, rudder, aileron, spoiler or fore plane, or a secondary flight control, such as a leading edge slat, trailing edge flap, air intake or airbrake, some means of moving the surface is necessary. Performance of the actuator can have a significant influence on overall aircraft performance and the implications of actuator performance on aircraft control at all operating conditions must be considered during flight-control system design and development programs.

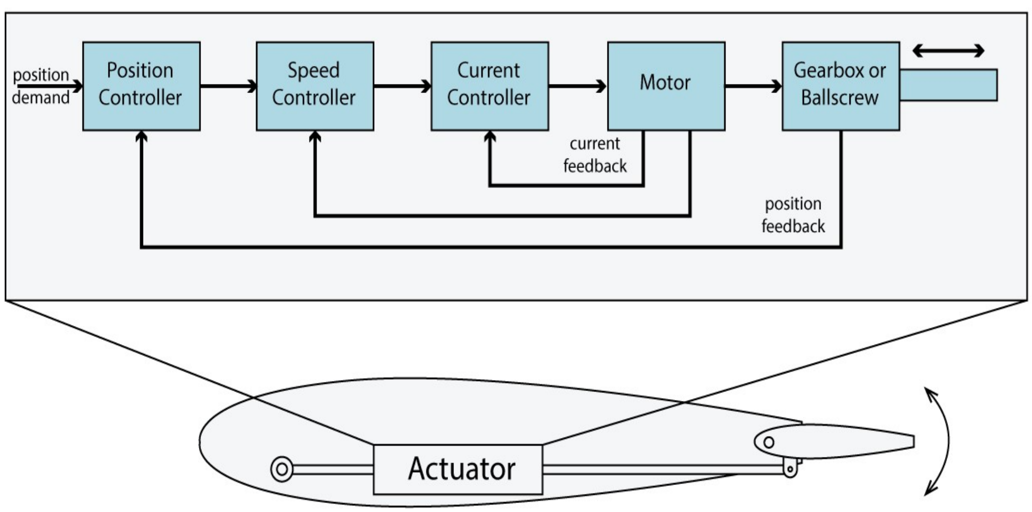

Figure 3.11 Actuation Systems

Overall aircraft performance requirements will dictate actuator performance requirements, which can lead to difficult design, control and manufacturing problems in their own right. An overview of current actuation system technologies as applied to modern combat aircraft is presented, and their performance and control requirements are discussed. The implications for aircraft control are considered and an overview of selected modelling and analysis methods is presented.

3.12 INTRODUCTION OF AIRCRAFT FLIGHT INSTRUMENTS

3.12.1 AIRSPEED INDICATOR

This device measures the difference between STATIC pressure (usually from a sensor not in the air-stream) and IMPACT or stagnation pressure from an aircraft's PITOT TUBE which is in the air-stream. During flight greater pressure will be indicated by PITOT TUBE and this difference in pressure from the static sensor can be used to calculate the airspeed.

$$V = \sqrt{pstg - pstat}/\rho \qquad \ldots\ldots..(4)$$

Figure 3.12.1 Airspeed Indicators

Primary Flight Group Instruments: Airspeed Indicator, Rate of Climb, Altimeter Linkages and Gears are designed to multiply the movement of the Diaphragm & provide indication on the dial of the Instrument. Instrument measures differential pressure between inside of diaphragm and instrument case.

True Airspeed

Adjusts the IAS for the given temperature and pressure. The F-15E receives TAS from the Air Data Computer which measures the outside temperature & pressure. True airspeed is calculated incorporating pressure and temperature corrections corresponding to flight altitude.

$$VT = Vi\sqrt{Pstd\,Tactual + Tstd\,Pactual} \qquad \ldots\ldots.(5)$$

VT= True airspeed, V= Indicated airspeed, p & T are pressure and temperature with subscripts and actual indicating standard and actual (altitude / ambient) conditions True Air Speed and Ground Speed will be the same in a perfectly still air.

Ground Speed

It is another important airspeed to pilots. Ground-speed is the aircraft's actual speed across the earth. It equals the TAS plus or minus the wind factor. For example, if your TAS is 500 MPH and you have a direct (180 degrees from your heading) tail-wind of 100 MPH, your ground-speed is 600 MPH. Ground-peed can be measured by onboard Inertial Navigation Systems (INS) or by Global Positioning Satellite (GPS) receivers. One "old-fashion" method is to record the time it takes to fly between two known points. Then divide this time by the distance. For example, if the distance is 18 miles, and it took an aircrew in an F-15E 2 minutes to fly between the points, then their ground-speed is:
18 miles / 2 minutes = 9 miles per minute.

3.12.2 ALTIMETER

It is one of the most important instruments especially while flying in conditions of poor visibility. Altitude must be known for calculating other key parameters such as engine power, airspeed etc. Altimeter works on the principle of barometer. In a sensitive altimeter there are three diaphragms capsule with two or three different dials each indicating different slab of altitude. Altimeter should be compensated for atmosphere pressure change.

Figure 3.12.2 Altitude Indicators

Altimeter senses normal decrease in air pressure that accompanies an increase in altitude. The airtight instrument case is vented to the static port. With an increase in altitude, the air pressure within the case decreases and a sealed aneroid barometer (bellows) within the case expands. The barometer movement is transferred to the indicator, calibrated in feet and displayed with two or three pointers. Different types of indicators display indicated altitude in a variety of ways,

Altitude Definitions

1. Indicated altitude is read directly from the altimeter when set to current barometric pressure.
2. Pressure altitude is read from the altimeter when set to the standard barometric pressure of 29.92 in. Hg.
3. Density altitude is the pressure altitude corrected for non- standard temperature.
4. True altitude is the exact height above mean sea level.
5. Absolute altitude is the actual height above the earth's surface.

3.12.3 RATE OF CLIMB METER

This is also called vertical speed indicator which is again useful in blind flights.

Level flights could be indicated by keeping the pointer on zero and subsequent changes are indicated in terms of ft. /minute.

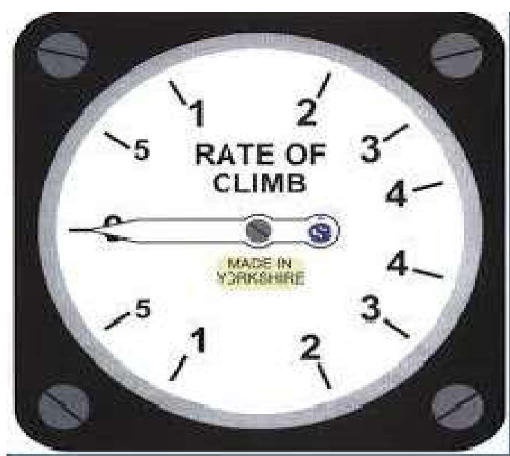

Figure 3.12.3 Climb Rate Indicators

This is also differential-pressure instrument -atmosphere and chamber pressure which is vented through a small capillary. Response of VSI is rather sluggish and is also sensitive to temperature changes. Mechanical stops prevent damage due to steep dives or maneuvers.

3.12.4 VERTICAL SPEED INDICATOR

Vertical Speed Indicator (VSI) displays vertical component of an aircraft's flight path. It measures the rate of change of static pressure in terms of feet per minute of climb or descent. VSI compensates for changes in atmospheric density. VSI is in a sealed case connected to the static line through a calibrated leak (restricted diffuser).

Figure 3.12.4 Vertical Speed Indicators

Diaphragm attached to the pointer by a system of linkages is vented to the static line without restrictions. With climb, the diaphragm contracts and the pressure drop faster than case pressure can escape through restructure, resulting in climb indications.

3.12.5 PITOT TUBE USE

Aircraft constantly encounter atmosphere pressure changes as they climb, descend, accelerate or decelerate. The pitot-static system - sensitive to airspeed, altitude, and rates of altitude change - provides the pressure information displayed on cabin instrumentation. An outside air temperature sensor must be installed for air data systems. The airspeed indicator is vented to both pitot and static lines. The airspeed indicator reacts to changes between pitot air and static air. The altimeter and vertical speed indicator, however, require venting to only the static line. Heated pitot tube prevents ice formation.

3.12.6 LOCATING PITOT

The static line vents the pitot-static instruments to the outside, or ambient, air pressure through the static port. The static port (may be located in various places on different types of aircraft and more than one port may be used. Regardless of location, the port is always positioned so the plane of the opening is parallel to the relative air flow.

Figure 3.12.6 Location of Pitot tube

By comparison, the plane of the pitot tube opening is nearly perpendicular to the relative wind. The pressure sensed at the static ports is transferred to the cabin instruments by a tube.

3.12.7 PITOT TUBE

Pitot tube on the aircraft is around 25 centimeters long with a 1 centimeter diameter. Several small holes are drilled around the outside of the tube and a center hole is drilled down the axis of the tube. The outside holes are connected to one side of a device called a pressure transducer. The center hole in the tube is kept separate from the outside holes and is connected to the other side of the transducer.

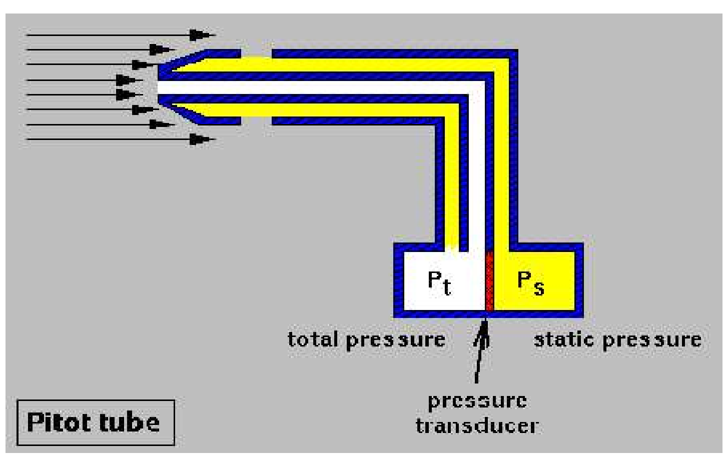

Figure 3.12.7 Pitot tube

The transducer measures the difference in pressure in the two groups of tubes by measuring the strain in a thin element using an electronic strain gauge. The pitot tube is mounted on the aircraft so that the center tube is always pointed in the direction of travel and the outside holes are perpendicular to the center tube Since the outside holes are Perpendicular to the direction of travel, these tubes are pressurized by the local random component of the air velocity. The pressure in these tubes is the static pressure (ps) discussed in Bernoulli's equation. The center tube, however, is pointed in the direction of travel and is pressurized by both the random and the ordered air velocity. The pressure in this tube is the total pressure (pt) discussed in Bernoulli's equation. The pressure transducer measures the difference in total and static pressure. (pt - ps). Some practical limitations:

1. If the velocity is low, the difference in pressures is very small and hard to accurately measure with the transducer. Errors in the instrument could be greater than measurement! So pitot tubes don't work very well for very low velocities.

2. If the velocity is very high (supersonic), we've violated the assumptions of Bernoulli's equation and the measurement is wrong again. At the front of the tube, a shock wave appears that will change the total pressure. There are corrections for the shock wave that can be applied to allow us to use pitot tubes for high speed aircraft.

3.13 TAKE – OFF AN AIRCRAFT

The take-off segment of an aircraft trajectory is shown in Fig.3.13.The aircraft is accelerated at constant power setting and at a constant angle of attack (all wheels on the ground) from rest to the rotation speed VR. For safety purposes, the rotation speed is required to be somewhat greater than the stall speed, and it is taken here to be

$$V_R = 1.2V_{stall}$$

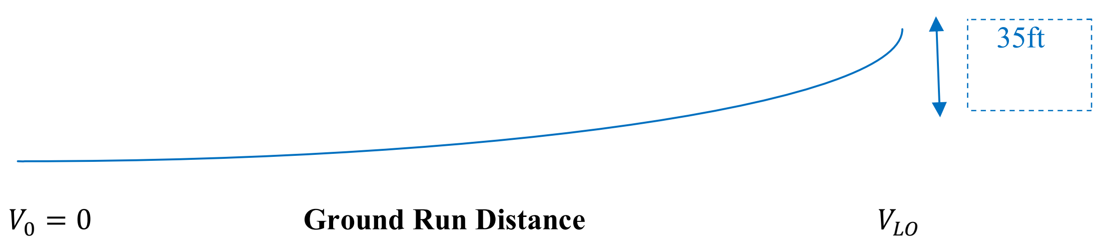

Figure 3.13 Take off of an aircraft

When the rotation speed is reached, the aircraft is rotated over a short time to an angle of attack which enables it to leave the ground at the lift-off speed VLO and begin to climb. The transition is also flown at constant angle of attack and power setting. The take-off segment ends when the aircraft reaches an altitude of h = 35 ft. Because airplanes are designed essentially for efficient cruise, they are designed aerodynamically for high lift-to-drag ratio. A trade- off is that the maximum lift coefficient decreases as the lift-to-drag ratio increases. This in turn increases the stall speed, increases the rotation speed, and increases the take-off distance. Keeping the take-off distance within the bounds of existing runway lengths is a prime consideration in selecting the size (maximum thrust) of the engines. The same problem occurs on landing but is addressed by using flaps. A low flap deflection can be used on take-off to reduce the take-off distance.

3.14 LANDING OF AN AIRCRAFT

The landing segment of an aircraft trajectory is shown in Fig.3.14. Landing begins with the aircraft in a reduced power setting descent at an altitude of h = 50 ft with gear and flaps down. As the aircraft nears the ground, it is flared to rotate the velocity vector parallel to the ground. The aircraft touches down on the main gear and is rotated downward to put the nose gear on the ground. Then, brakes and sometimes reverse thrust, spoilers, and a drag chute are used to stop the airplane. The landing ends when the aircraft comes to rest. For safety purposes, the touchdown speed is required to be somewhat greater than the stall speed and is taken here to be

$$VTD = 1.2Vstall$$

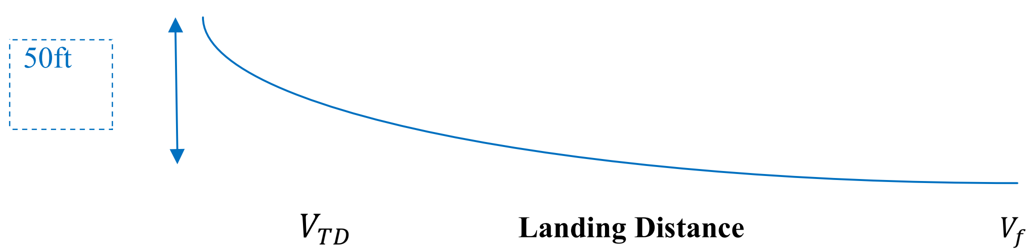

Figure 3.14 Landing of an aircraft

3.15 EQUATIONS OF MOTION

The term flight mechanics refers to the analysis of airplane motion using Newton’s laws. While most aircraft structures are flexible to some extent, the airplane is assumed here to be a rigid body. When fuel is being consumed, the airplane is a variable-mass rigid body. Newton's laws are valid when written relative to an inertial reference frame, that is, a reference frame which is not accelerating or rotating. If the equations of motion are derived relative to an inertial reference frame and if approximations characteristic of airplane motion are introduced into these equations, the resulting equations are those for flight over a non-rotating flat earth. Hence, for airplane motion, the earth is an approximate inertial reference frame, and this model is called the flat earth model. The use of this physical model leads to a small error in most analyses.

A general derivation of the equations of motion involves the use of a material system involving both solid and fluid particles. The end result is a set of equations giving the motion of the solid part of the airplane subject to aerodynamic, propulsive and gravitational forces. Introduction to Airplane Flight Mechanics for the forces are assumed to be known. Then, the equations describing the motion of the solid part of the airplane are derived. The airplane is assumed to have a right-left plane of symmetry with the forces acting at the center of gravity and the moments acting about the center of gravity. Actually, the forces acting on an airplane in fight are due to distributed surface forces and body forces. The surface forces come from the air moving over the airplane and through the propulsion system, while the body forces are due to gravitational effects. Any distributed force can be replaced by concentrated force acting along a specific line of action. Then, to have all forces acting through the same point, the concentrated force can be replaced by the same force acting at the point of interest plus a moment about that point to offset the effect of moving the force. The point usually chosen for this purpose is the center of mass, or equivalently for airplanes the center of gravity, because the equations of motion are the simplest. The equations governing the translational and rotational motion of an airplane are the following:

a. Kinematic equations giving the translational position and rotational position relative to the earth reference frame.

b. Dynamic equations relating forces to translational acceleration and moments to rotational acceleration.

c. Equations defining the variable-mass characteristics of the airplane (center of gravity, mass and moments of inertia) versus time.

d. Equations giving the positions of control surfaces and other movable parts of the airplane (landing gear, flaps, wing sweep, etc.) versus time.

These equations are referred to as the six degree of freedom (6DOF) equations of motion. The use of these equations depends on the particular area of flight mechanics being investigated.

3.16 ANGLE OF ATTACK

Angle of attack is the angle at which the oncoming air meets the wing. The lift of an airplane is the lift of the wing-body combination plus the lift of the horizontal tail. The lift of a wing-body combination is a complicated affair in that the body produces some lift and interference effects between the wing and the body increase the lift of the body. It has been observed that the lift of a wing-body combination can be replaced by the lift of the entire wing (including that portion which passes through the fuselage). The lift of the horizontal tail is neglected with respect to that of the wing. Hence, the lift of the airplane is approximated by the lift of the entire wing. Geometrically, the wing is defined by its plan form shape, its airfoil shapes along the span, and the shape of its chord surface. The only wings considered here are those with a straight- tapered plan form shape, the same airfoil shape along the span, and a planar chord surface (no bend or twist). If a wing does not meet these conditions, it can be replaced by an average wing that does. For example, if the airfoil has a higher thickness ratio at the root than it does at the tip; an average thickness can be used. The aerodynamic characteristics of airfoils and wings have been taken. Over the range of lift coefficients where aircraft normally operate, the lift coefficient of the wing can be assumed to be linear in the angle of attack that is,

$$CL = CL\alpha(\alpha - \alpha_0 L) \qquad \text{……….} (6)$$

Where $\alpha_0 L$ is the zero-lift angle of attack and CLα is the lift-curve slope of the wing. This equation can be solved for α as

$$\alpha = \alpha_0 L(M) + (CL/CL\alpha(M)) \qquad \text{………..} (7)$$

Hence, to obtain α, it is necessary to determine α0L and$CL\alpha$. First, Air foils are discussed, then wings, then airplanes.

3.17 SIX DEGREE OF FREEDOM (6-DOF) MODEL: WIND AXES

The translational equations have been uncoupled from the rotational equations by assuming that the aircraft is not rotating and that control surface deflections do not affect the aerodynamic forces. The scalar equations of motion for flight in a vertical plane have been derived in the wind axes system. These equations have been used to study aircraft trajectories (performance). If desired, the elevator deflection history required by the airplane to fly a particular trajectory can be obtained by using the rotational equation. In this chapter, the six-degree-of-freedom (6DOF) model for non-steady flight in a vertical plane is presented in the wind axes system. Formulas are derived for calculating the forces and moments. Because it is possible to do so, the effect of elevator deflection on the lift is included. These results will be used in the next chapter to compute the elevator deflection required for a given flight condition. Finally, since the equations for the aerodynamic pitching moment are now available, the formula for the drag polar can be improved by using the trimmed polar.

3.18 CRITICAL SITUATIONS IN TAKE OFF AND LANDING FLIGHT PHASES

Figure 3.18 Takeoffs and Landing Phase of Aircraft

A critical situation during the takeoff phase or a landing phase could be an engine out condition. In this case, the operative engine will create a force moment that has to be balanced by a side aerodynamic force created by the rudder deflection. In a normal airplane landing the vertical speed towards the ground is about 2 to 4 m/s. If the vertical speed is between 6 m/s and 8 m/s, we have a hard landing, and the problem just a matter of a control maintenance of the landing gear. If the landing vertical speed is higher than 8m/s, we have a crash problem occur. This situation can happen because pilot error in landing procedures (vertical speed too high or not the correct position of the plane with respect to the ground), special meteorological phenomena, as turbulence (vertical speed towards the ground) or wind shear (wind velocities parallel to the ground, that decrease suddenly the relative on the speed of the airplane wind reference to the air). Sometimes it’s occurring due to incorrect reading of the control instruments. Flight control problems include gross weight and center-of-gravity problems, jammed or locked controls, aircraft stall, instrument error or false indications (like airspeed indicator). Airspeed Indicator creates Problems when stop working. Basically at taxiing and taking off the speed indicator is works fine. When aircraft in the air it sometime stop working. This situation is very critical for a pilot.

A review of some of the general aviation reports seems to indicate that pilot error in responding to the situation caused more of a problem than the electrical problem. Because many of the reports had little or no damage reported, the narrative of the reports were very brief without a lot of details. For example, one report about a Cessna 182 stated, Electrical problem, Alternator field wire loose. In confusion landed gear up." Again, minor damage was done to the aircraft. Another pilot while descending from altitude did a "long cruise descent with the engines at a very low power output. The aircraft had generators instead of alternators, and that the engine speed was using for the descent was below the speed required to keep the battery charged." After landing the commercial pilot and flight instructor discovered the aircraft's battery was too low to start the aircraft.

CHAPTER 4

ORDER REDUCTION OF AIRCRAFT PITCH MODEL BY PROPOSED METHODS

4.1 PROPOSED MODEL

In model order reduction, we try to solve the problem of finding the reduced order model of an original higher order model, with the minimum error and the best cost. Our goal is to find another transfer function which describe the same system but with less data, *i.e*. lower order. In next section, we will find out how we can use ROUTH ARRAY and PADE APPROXIMATION methods in MOR. In Model Order Reduction problem, the solution that the ROUTH ARRAY and PADE APPROXIMATION will try to find the aircraft pitch coefficients of the transfer functions.

4.1.1 AIRCRAFT PITCH MODEL

The equations governing the motion of an aircraft are a very complicated set of six nonlinear coupled differential equations. However, under certain assumptions, they can be decoupled and linearized into longitudinal and lateral equations. Aircraft pitch is governed by the longitudinal dynamics. In this example we will design an autopilot that controls the pitch of an aircraft.

The basic coordinate axes and forces acting on an aircraft are shown in the figure given below.

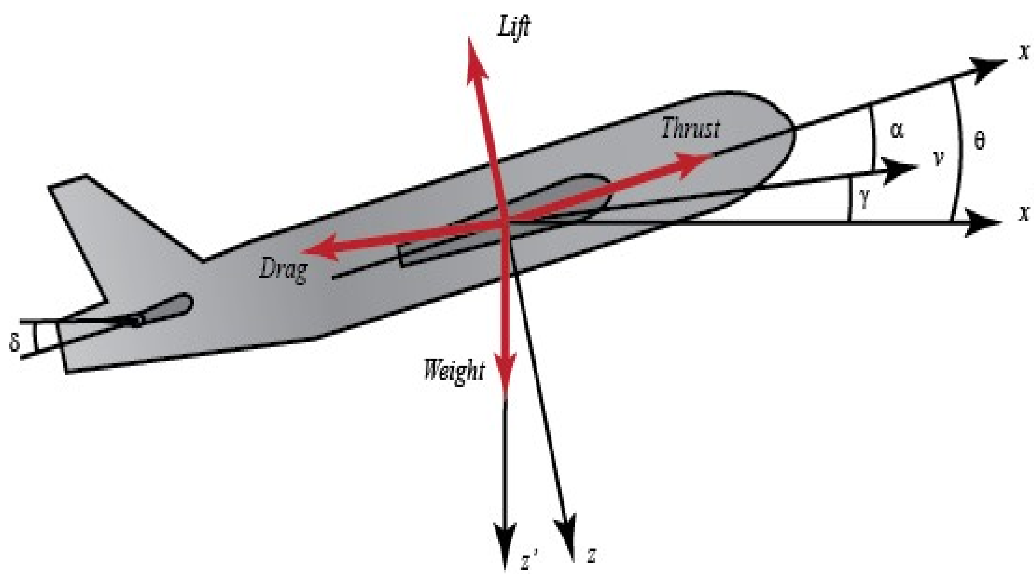

Figure 4.1.1 Aircraft Pitch Model

We will assume that the aircraft is in steady-cruise at constant altitude and velocity; thus, the thrust, drag, weight and lift forces balance each other in the *x*- and *y*-directions. We will also assume that a change in pitch angle will not change the speed of the aircraft under any circumstance (unrealistic but simplifies the problem a bit). Under these assumptions, the longitudinal equations of motion for the aircraft can be written as follows.

$$\alpha = \mu\Omega\sigma[-(C_L + C_D)\alpha + \frac{1}{(\mu - C_L)}q - (C_W \sin\gamma)\theta + C_L] \quad \text{……… (8)}$$

$$q = \frac{\mu\Omega}{2i_{yy}} \; [[C_M - \eta(C_L + C_D)]\alpha + [C_M + \sigma C_M(1 - \mu C_L)]q + (\eta C_W \sin\gamma)\delta \;]$$

$$\theta = \Omega q \quad \text{…….. (9)}$$

Where,

$\alpha = Angle\ of\ attack$

$q = Pitch\ rate$

$\theta = Pitch\ angle$

$\delta = Elevator\ deflection\ angl$

$$\mu = \frac{\rho S\bar{e}}{4m}$$

Where,

$\rho = Density\ of\ air$

$S = Platform\ area\ of\ the\ wing$

$\bar{e} = Average\ chord\ length$

$m = Mass\ of\ the\ aircraft$

$$\Omega = \frac{2U}{\bar{e}}$$

Where,

$U = Equilibrium\ flight\ speed$

$C_T = Coefficient\ of\ Thrust$

$C_D = Coefficient\ of\ drag$

$C_L = Cofficient\ of\ lift$

$C_W = Coefficient\ of\ weight$

$C_M = Coefficient\ of\ pitch\ moment$

$\gamma = Flight\ Path\ angle$

$$\sigma = \frac{1}{1 + \mu C_L} = Constant$$

I_{yy} = Normalized moment of intertia

$$\eta = \mu_0 C_M = Constant$$

Before finding the transfer function and state-space models, let's plug in some numerical values to simplify the modeling equations shown above:

$$\alpha = -0.313\alpha + 56.7q + 0.232\delta \qquad \text{...... (10)}$$

$$q = -0.0139\alpha - 0.426q + 0.0203\delta \qquad \text{....... (11)}$$

$$\theta = 56.7q \qquad \text{....... (12)}$$

These values are taken from the data from one of Boeing's commercial aircraft.

4.1.2. Transfer function

To find the transfer function of the above system, we need to take the Laplace transform of the above modeling equations. Recall that when finding a transfer function, zero initial conditions must be assumed. The Laplace transform of the above equations are shown below.

$$s\alpha(s) = -0.313A(s) + 56.7Q(s) + 0.232\Delta(s) \qquad \text{....... (13)}$$

$$sq(s) = -0.0139A(s) - 0.426Q(s) + 0.0232\Delta(s) \qquad \text{...... (14)}$$

$$s\theta(s) = 56.7Q(s) \qquad \text{....... (15)}$$

After few steps of algebra, you should obtain the following transfer function.

$$G(s) = \frac{\theta(s)}{\Delta(s)} = \frac{1.151s + \ .1774}{s^3 + 0.739s^2 + 0.921s} \qquad \text{........ (16)}$$

Matlab Coding of original system

- **n=[1.151 0.1774];**
- **d=[1 0.739 0.921 0];**
- **tf(n,d)**
- **step();**

4.2 PROPOSED METHODS

4.2.1 ROUTH ARRAY

In Routh approximation, the reciprocals of the coefficients for both numerator and denominator polynomial's alpha (for denominator) and beta (for numerator) tables.The reduced order model is determined and reconsidered for reciprocal of their coefficients to calculate final decreased model using Routh approximation method [3].The application of model order reduction techniques have been considered for reduction of single-machine infinite-bus (SMIB)power system in [3]. The Routh stability array method is based on array method, in which; array for numerator and denominator polynomials are derived. In stability equation method, the reduced models with a successively elimination of two high-order elements in each step are obtained to get the low order system [16]. The application of Routh stability array method is presented in [3]. This method uses the generation of Routh array by using coefficients of given *nth* high-order polynomial of a problem. First, two rows indicate generated rows, having the coefficients of original HOS. After that all the rows known as computed rows derived from previous

Table 1. Generating Rows

a_0	a_1	a_2	a_3	_ _ _
b_0	b_1	b_2	b_3	_ _ _

Table 2. Computing Rows

c_0	c_1	c_2	c_3	_ _ _
d_0	d_1	d_2	d_3	_ _ _

two rows. First row of generated rows indicates 1*st*, 3*rd*, 5*th*, *:::* order coefficients and second rows indicate 2*nd*, 4*th*, 6*th*, *:::* order coefficients. Consider an nth order polynomial HOS $P(s)$ is given below as in Eq. 10

$$P(s) = a_0 s^n + b_0 s^{(n-1)} + a_1 s^{(n-2)} + b_1 s^{(n-3)} + \cdots \qquad \text{…….} (17)$$

Routh Array method is popular for determining the stability of high order polynomial system. Above given Table 1, which indicates generated rows easily understandable by Eq.(15),

Whereas Table 2 indicates computed rows explained below by mathematical procedure used in Eq. (12)

$c_0 = \frac{b_0 a_1 - a_0 b_1}{b_0}$ $\qquad$ $c_1 = \frac{b_0 a_2 - a_0 b_2}{b_0}$

……. (18)

$d_0 = \frac{c_0 b_1 - b_0 c_1}{c_0}$ $\qquad$ $d_1 = \frac{c_0 b_2 - b_0 c_2}{c_0}$

Where $a0$, $a1$, _ _ _ and $b0$, $b1$, _ _ _ etc. are the coefficients of generated rows of original system.
And $d0$, $d1$, _ _ _ and $c0$, $c1$, _ _ _ are the coefficients of computed rows derived from just previous two rows.

Here numerator is first order equation. So not reducing the numerator and reduction process is applied on denominator.

Denominator is – $s^3 + 0.739s^2 + 0.921s$
By routh array table

s^3	1	0.921
s^2	0.739	0
s^1	0.921	

Numerator is 1.15s+0.1774
And Denominator is $s^3 + 0.739s^2 + 0.921s$

Transfer function by Routh Array is

$$T.F = \frac{1.151s+0.1774}{0.739s^2+0.921s} \qquad …….. (19)$$

Matlab Coding of Routh Array

- **n=[1.151 0.1774];**
- **d=[0.739 0.921 0.00045];**
- **s=tf (n,d)**
- **step(s);**

4.2.2 Pade Approximation

Pade introduced this technique and Shamash applied this method. This method is computationally simple, fits initial time moments and steady state value of original and reduced order model matches. For rth order model, '2r-1' coefficients of power series expansion (about s=0) of reduced order model matched with the corresponding coefficients of the original system. The disadvantage this method is that reduced order model may be unstable even though the original system is stable. Also it may sometimes approximate non –dominant poles of the system, thus giving bad approximation. To overcome this disadvantage, various alternatives methods have been suggested. Shamash introduced a method of reduction based on retention of poles of high order system in reduced order model and concept of Pade approximation about more than one point.

Let the transfer function of high order original system of the order 'n' be

$$G(s) = \frac{N(s)}{D(s)} = \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_{n-1} s^{n-1}}{b_0 + b_1 s + b_2 s^2 + \cdots + b_n s^n} \quad \ldots \ldots (20)$$

Where; a_i $0 \leq i \leq n-1$ and b_i $0 \leq i \leq n$ are known scalar constant.

Reduced transfer function of original model:

$$R_k = \frac{N(s)}{D(s)} = \frac{c_0 + c_1 s + c_2 s^2 + \cdots + c_{k-1} s^{k-1}}{d_0 + d_1 s + d_2 s^2 + \cdots + d_k s^k} \quad \ldots \ldots (21)$$

Where, c_i $0 \leq i \leq k-1$ and d_i $0 \leq i \leq k$ are known scalar constant.

The objective of this paper is to realize the kth order reduced model in the form of (21) from the original system (20) such that it retains the important features of the original high order system.

The reduction procedure consists of the following steps:

Determination of the denominator coefficients of the reduced model by using the pade approximation.
The original nth -order system can be expanded in power series about s = 0 as

$$G_n(s) = \frac{\sum^{n-1}{}_0 a_i s}{\sum_0^n b_i s} = e_0 + e_1 s + e_2 s^2 + \cdots \quad \ldots \ldots (22)$$

The coefficients of the power series expansion can also be calculated as follows:

$$e_0 = \frac{a_0}{b_0} \quad \ldots \ldots (23)$$

$$e_1 = \frac{1}{b_0} \left[a_i - \sum_{j=1}^{1} a_j e_{i-j} \qquad i > 0 \right. \quad \ldots \ldots (24)$$

$$a_i = 0 \qquad i > n-1$$

The k^{th} - order reduced model is taken as

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{\sum_{i=0}^{k-1} c_i s^i}{\sum_{i=0}^{k} d_i s^i} \qquad \ldots \ldots (25)$$

For $N_k(s)$ of eq. (25) to be Pade approximants of Gn(s) of equation (22), we have

$$\left.\begin{array}{l} c_0 = d_0 e_0 \\ c_1 = d_0 e_1 + d_1 c_0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ c_{k-1} = d_0 e_{k-1} + d_1 e_{k-2} + \cdots + d_{k-2} e_1 + d_{k-1} e_0 \end{array}\right\} \qquad \ldots\ldots (26)$$

The coefficients cj; j=0, 1, 2.........*k*-1 can be found by solving the above *k* linear equations.

Hence, the numerator $N_k(s)$ is obtained as

$$N_k(s) = c_0 + c_1 s + c_2 s^2 + c_3 s^3 + \cdots + c_{k-1} s^{k-1} \qquad \ldots \ldots (27)$$

Here numerator is first order equation. So not reducing the numerator and reduction process is applied on denominator.

Denominator is $-s^3 + 0.739s^2 + 0.921s$

By Pade Approximation method

The coefficients of the power series expansion

$$e_0 = \frac{a_0}{b_0} = \frac{0.1774}{0.921} = 0.192$$

$$e_1 = \frac{1}{b_0}\left[a_i - \sum_{j=1}^{1} a_j e_{i-j}\right] \qquad i > 0$$

$$= \frac{1}{b_0}\left[a_1 - (a_1 e_0)\right]$$

$$= \frac{1}{0.921}[1.151 - (1.151 \times 0.192$$

$$= 1.0089$$

$$e_2 = \frac{1}{b_0}\left[a_2 - (a_1 e_1 + a_2 e_0)\right]$$

$$= \frac{1}{0.921}[0 - (1.151 \times 1.0089 + 0 \times 0.192)]$$

$= -1.2595$

For denominator

$c_0 = d_0 e_0 = 0.1774 \times 0.192 = 0.03406$

$c_1 = d_0 e_1 + d_1 c_0$
$= 0.1774 \times (-1.2595) + 1.151 \times 1.0089 + 0$
$= 0.93679$

Hence, the numerator $D_k(s)$ is obtained as

$D_k(s) = 0.03406 + 0.39977s + 0.93679s^2$

So the Transfer function by Pade Approximation Method is

$$T.F = \frac{N_k(s)}{D_k(s)} = \frac{0.1774 + 1.151s}{0.03406 + 0.39977s + 0.93679s^2} \quad \ldots\ldots\ldots(28)$$

Matlab Coding of Pade Approximation

- **n=[1.15 0.1774]**
- **d=[0.93679 0.39977 0.03406]**
- **ss=tf(n,d)**
- **step(ss,100);**
- **hold();**

CHAPTER 5

RESULT

5.1 RESULT OF AIRCRAFT PITCH MODEL WITHOUT APPLICATION OF TECHNIQUES

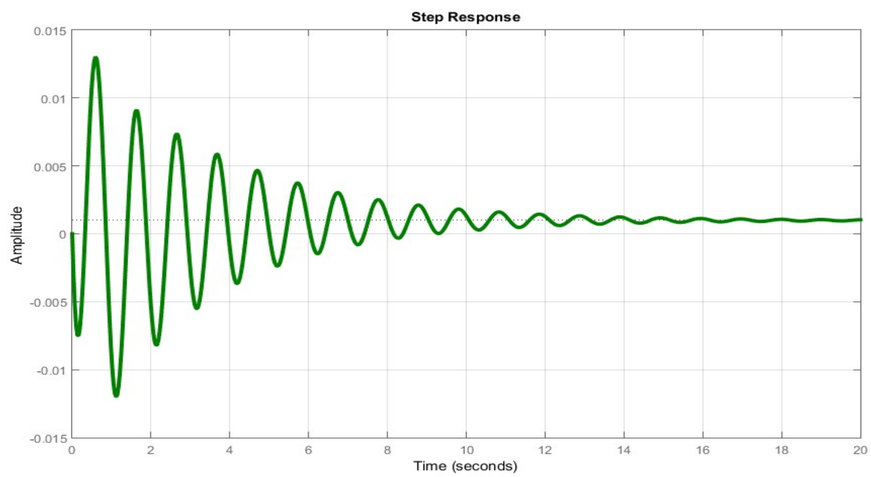

Figure 5.1 Result of Original system

5.2 RESULT OF AIRCRAFT PITCH MODEL WITH APPLICATION OF TECHNIQUES

5.2.1 Result of Routh Array Method

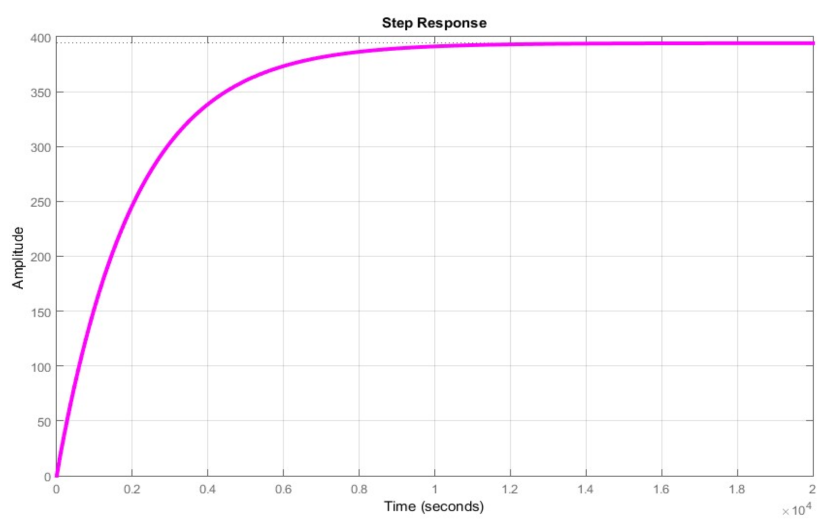

Figure 5.2.1 Result of Routh Array

5.2.2 Result of Pade Approximation Method

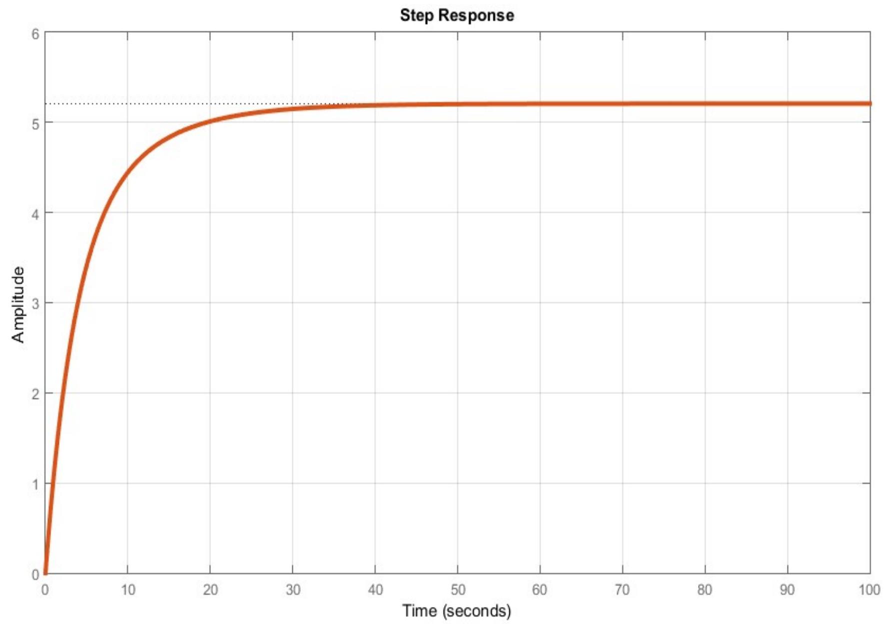

Figure 5.2.2 Result of Pade Approximation

5.3 TABLE OF COMPARISON ANALYSIS BETWEEN ROUTH ARRAY AND PDE APPROXIMATION METHOD FOR AIRCRAFT PITCH MODEL

S.No.	Method	Rise Time(sec)	Settling Time(sec)	Peak	Peak Time(sec)	Overshoot
1.	Original	0.0112	13.5	0.013	0.589	1.21e+03
2.	Routh	1.2	1.4	394	2e+03	0
3.	Pade	12.1	25.2	5.21	100	0

Figure 5.3 Table of comparison of Routh Array and Pade Approximation

5.4 ADVANTAGES

- High fidelity representation of the original large scale system.
- Considerable difference between the size of model order reduction n and original model.
- Small approximation error or global error bound.
- Preservation of system properties like stability /passivity.
- Numerically stable & efficient procedure.

5.5 APPLICATION

- Aero elastic flutter analysis.
- System modeling for active flow control.
- Electronic, fluid and structural mechanics.

CHAPTER 6

CONCLUSIO N AND FUTURE SCOPE

6.1 CONCLUSION

- As we see that the settling time for Aircraft Pitch Model without application of Intelligent technique is 13.5 sec and the value overshoot is 1.21 e+03 whereas the value of settling time and overshoot for Routh Array and Pade approximation techniques are 1.4, 25.2 and 0,0 respectively.

- In the tabular Analysis, we see that Routh Array reduces the value of settling time by 89.62% as compared original system.

- The value of overshoot comes to 0 in the case of Pade Approximation and Routh Array Method which is practically requirement of every system.

- The value of overshoot comes to 0 in the case of Pade Approximation and Routh Array Method which is practically requirement of every system.

6.2 SCOPE FOR FUTURE WORK

The work in this area can be extended in various directions. The methods proposed in fourth chapter can be extended for reduction of discrete time system. Criterions could be devised to select the order of reduced order model that accurately represent the original system by the proposed method. The reduction methods have been proposed by extension of SISO methods to reduce MIMO systems by considering each elements of the transfer function. This approach works well if there is a common denominator of the system, but if this is not the case then successive application SISO method to MIMO system may lead to a reduced system whose order may be equal or greater than the high order system in some cases. Therefore SISO methods can be extended for MIMO systems in such a way that all elements of the transfer matrix are handled simultaneously.

Chapter 4 deals with the various model order reduction techniques which are proposed. These methods can be utilized for order reduction of aircraft pitch model.

The chapter 3 considers the applications of reduction method for proposed model. Extensive investigation can be undertaking from application present overview. In this chapter the aircraft pitch transfer function reduced using approximate model matching technique. So some soft computing techniques such as GA, PSO etc can be used for determining the transfer function of model.

The recent trends in VLSI industry towards miniature designs, low power consumption, high speed digital circuits with increased integration of analog circuits with digital blocks have made the signal integrity analysis a challenging task. A lot of research work has to be under taken to device the model order reduction a viable tool in the field of linear time invariant high speed VLSI circuit and theory aspect.

CHAPTER 7

LITERATURE REFERENCES

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Babu Banarasi Das University

Plagiarism Report

Student Name – Priya Dhar

Roll no - 1170450004

Thesis title - Performance analysis of Aircraft Pitch Model using Routh Array and Pade Approximation Techniques.

Guide- Mr. Akash Varshney Assistant Professor

Plagiarism Report Details

- 85.50% unique content
- 14.50% plagiarism

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LIST OF PUBLICATION

- Priya Dhar, Akash Varshney, “Performance Analysis of Aircraft Pitch Model Using Routh Array and Pade Approximation Techniques” International Journal for Innovations in Engineering, Science and Management. ISSN 2347-7911, Volume 7, Issue 3, March 2019.

- Priya Dhar, Akash Varshney, “Comparative Analysis of Higher Order System through Routh Array and Pade Approximation Techniques” International Journal for Innovations in Engineering, Science and Management. ISSN 2347-7911, Volume 7, Issue 3, March 2019.

APPENDIX

BABU BANARASI DAS UNIVERSITY, LUCKNOW

CERTIFICATE OF FINAL THESIS SUBMISSION

1. Name: Priya Dhar
2. Roll No: 1170450004
3. Thesis Title: Performance analysis of Aircraft Pitch Model using Routh Array and Pade Approximation Techniques.
4. Degree for which thesis is submitted: Master of Technology (Power System & Control)
5. School (of the University to which the thesis is submitted)

School of Engineering

6. Thesis Preparation Guide was referred to for preparing the thesis. YES ☐ NO ☐
7. Specification regarding thesis format have been closely followed. YES ☐ NO ☐
8. The content of the thesis have been organized based on Guidelines YES ☐ NO ☐
9. The thesis has been prepared without resorting to plagiarism. YES ☐ NO ☐
10. All sources used have been cited appropriately. YES ☐ NO ☐
11. The thesis has not been submitted elsewhere for a degree. YES ☐ NO ☐
12. All the corrections have been incorporated YES ☐ NO ☐
13. Submitted 4 hard bound copies plus one CD. YES ☐ NO ☐

(Signature of the Supervisor)
Name: Mr. Akash Varshney

(Signature of the candidate)
Name: Priya Dhar
Roll No: 1170450004

CURRICULUM VITAE

PRIYA DHAR

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Email ID – pdshakti@gmail.com

Career Objectives

Looking for the opportunity to the make contribution of my professional knowledge at a right place and also to enhance my skills enabling to further upgrade my contribution to the organization.

Technical Qualification

- M.Tech. in Power System and Control, from BBD University Lucknow.
- B.Tech. in Electrical Engineering with 66.2 %, from BBD University Lucknow.
- Diploma in Electrical Engineering with 72.9 %, from M.P.Polytechnique Gorakhpur.

Academic Profile

- Intermediate Passed from UP Board with 57.60%
- High School Passed from UP Board with 60.17%

Technical Skills

- Language-C
- Software Platform - Matlab 2013, 2015, MS-Office
- Operating System – Window -7/8/10

Training Certificate

- Completed four weeks Vocational industrial training from NTPC Power Plant Faizabad UP.

Extra Activities

- NCC “B” Certificate
- P.G.D.C.A

Inter Personal Skills

- Ability to work in team
- Loyal and self Motivated
- Hard working
- Willing to learn and adapt to new opportunities and challenges

Personal Details

- Father's Name : Mr. Shakti Dhar
- Mother's Name : Mrs. Bindu Mat
- Languages Known : Hindi & English

Declaration

I hereby declare that the above information given in this resume is correct up to best of my knowledge and I will be solely responsible for any discrepancy.

Date-

Place- **(Priya Dhar)**