PERFORMANCE ANALYSIS OF WIRELESS NETWORK IN α-μ FADING CHANNEL

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By

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CERTIFICATE

DECLARATION BY THE CANDIDATE

I, hereby, declare that the work presented in this thesis, entitled "**Performance Analysis of Wireless Network in *α-μ* Fading Channel"** in fulfilment of the requirements for the award of Degree of Doctor of Philosophy of Babu Banarasi Das University, Lucknow is an authentic record of my own research work carried out under the supervision of **Dr. Himanshu Katiyar, Associate Professor, BBDNIIT**.

All the work have been published in international journals and conferences during this Ph.D. program. I also declare that the work embodied in the present thesis is my original work and has not been submitted by me for any other Degree or Diploma of any university or institution.

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CERTIFICATE OF THE SUPERVISOR

This is to certify that the thesis, entitled "**Performance Analysis of Wireless Network in α-μ Fading Channel"** submitted by **Sh Dharmraj** for the award of Degree of Doctor Philosophy by Babu Banarasi Das University, Lucknow is a record of authentic work carried out by him under my supervision. To the best of my knowledge, the matter embodied in this thesis is the original work of the candidate and has not been submitted elsewhere for the award of any other degree or diploma.

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Preface

In the past decade, α-μ fading channel has emerged as an attractive, flexible and mathematically easily tractable wireless channel model for multipath fading variations because it includes important channel distributions such as Gamma, Nakagami-m, Exponential, Weibull, One-sided Gaussian, and Rayleigh. An attempt has been made in this thesis report to analyse the performance metrics, outage and bit error rate over a-μ fading channel in different scenarios such as diversity, correlation, channel coding, dual-hop/ Multi-hop relaying and cooperative relaying.

In recent years, a new paradigm for communication called cooperative communications has been reported for which initial studies have shown the potential for improvements in capacity over traditional multi-hop wireless networks. The basic concept of cooperative communication/relay networks is to utilize the relay nodes for conveying the source's messages to the destination. The transmission between the source and destination nodes is divided into two main phases; Broadcasting phase - the source transmits its messages to both relay and destination, and Multiple-access phase - the relay manipulates its received messages from the source before forwarding them to the destination. With this transmission strategy, cooperative communications overcomes the severe path loss and shadowing effects, and exhibits remarkable improvement in performance. The cooperative communications achieves significant power savings, expands the communication range, and keep the implementation simple.

Cooperative communication over α-μ fading channel is a new research area, and we have not come across any research paper on this topic during our extensive literature survey. The α-μ fading channel has been simulated in MATLAB by using the algorithm stated in appendix B. It is seen that the analytical and simulated results match with each other. Most of the work carried out during this doctoral course has been published in refereed journals or presented in peer-reviewed national & international conferences/seminars and received due appreciation. Details are as given below:

Journal Publications:

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2. Dharmraj and Katiyar Himanshu, *"Bit Error Rate Analysis of Block Coded Wireless System over α-μ Fading Channel*", International Journal of Advance Research in Computer and Communication Engineering (IJARCCE), ISSN 2378-1021, vol.4, iss.8, Aug 2015, pp.165-169. (*DOI 10.17148/IJARCCE.2015.4834*).

3. Dharmraj and Katiyar Himanshu, *"Error Analysis of Multi-User CDMA System over α-μ Fading Channel*", International Journal of Advance Research in Computer and Communication Engineering (IJARCCE), ISSN 2378-1021, vol.4, iss.10, Oct 2015, pp.533-537. (*DOI 10.17148/IJARCCE.2015.410121*).

4. Dharmraj and Katiyar Himanshu, *"Performance Analysis of Space Diversity in α-μ Fading Channel*", International Journal of Electrical and Electronics Engineers (IJEEE), ISSN 2321-2055, vol. 7, iss. 2, Jul - Dec 2015, pp.38-46.

5. Dharmraj and Katiyar Himanshu, *"Performance Analysis of Dual Diversity Receiver over Correlated α-μ Fading Channel*", International Journal of Advance Research in Science and Engineering (IJARSE), ISSN 2319-8354, vol. 4, iss. 8, Aug 2015, pp.92-100.

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7. Dharmraj and Katiyar Himanshu, *"Comparative Performance of Two-hop Relay System over α-μ Fading Channel*", under revision in the Springer, The Institution of Engineers (India) B series. (IEIB-S-15-00483)

8. Dharmraj and Katiyar Himanshu, *"Optimal Power Allocation for Regenerative Relay over α-μ Fading Channel*", International Journal of Computer and Communication Technologies (IJCCTS), ISSN 2278-9723, vol.3, no.11, Sep.2015, pp.761-766.

9. Dharmraj and Katiyar Himanshu, *"Performance Analysis of Multi-Hop Relay-Network over α-μ Fading Channel*", International Journal of Research in Electronics and Communication Technology (IJRECT), ISSN 2348-9065, vol.2, iss.3, Jul-Sep 2015, pp.22-28.

10. Dharmraj and Katiyar Himanshu, *"Performance Analysis of Cooperative Relaying over α-μ Fading Channel*", International Journal of Innovative Research in Computer and Communication Engineering (IJIRCCE), ISSN 2320-9801, vol.3, iss.8, Aug. 2015, pp.7204-7213. (*DOI: 10.15680/IJIRCCE.2015. 0308115*)

Conferences/ Seminars Publications:

1. Dharmraj, Rastogi Ashutosh, and Katiyar Himanshu, *"Overview of Diversity Combining Technique & Cooperative Relaying in Wireless Communication*", Proceedings of 'International Symposium on Recent Trends in Electronics and Communication (ISRTEC)', organized by KNIT Sultanpur, on 8-9 Nov 2012.

2. Dharmraj, and Katiyar Himanshu, *"Generalized Fading Channels in Wireless Communication"*, Proceedings of The Institution of Electronics and Telecommunication (IETE) Lucknow Seminar on "Rural Empowerment through Broadband" 16-17 Mar 2013.

3. Dharmraj, and Katiyar Himanshu, *“Generalized Fading Channels in Wireless Communication”,* Poster Presentation in 1st BBDU Scholars Conclave 12 Jul 2013.

4. Dharmraj, and Katiyar Himanshu, *“Frugal Innovation in ICT (Information and Communication Technologies) in India”* published in Souvenir on 46th Engineers Day, The Institution of Engineers (India), Lucknow on 15 Sep 2013.

5. Dharmraj, and Katiyar Himanshu, *“Diversity Combining and α-μ Generalized Fading Model in Wireless Communication”*, Presented in Technical Session of Annual General Meeting (AGM) of The Institution of Engineers (India) Lucknow on 26 Oct 2013.

   The paper was judged in best paper category and received **“The Institution Award”** along with **Cash Prize of Rs. 2500**.

6. Dharmraj, and Katiyar Himanshu, *“The Alpha-mu Distribution: A Generalized Fading Distribution Model in Wireless Communication*”, Proceedings of 2nd National Conference on ‘Challenges and Opportunities for Technological Innovation in India COTII-2014’, organized by AIMT Lucknow on 22 Feb 2014.

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1. **NPTEL Course: IIT Kanpur**

   Registered (NOC15EC051330011) in National Program on Technology Enhanced Learning (NPTEL) two months (Jul–Aug 2015) course on *“Principles of Modern CDMA / MIMO / OFDM Wireless Communication”* conducted by Prof. Aditya K. Jagannatham, IIT Kanpur.
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Abstract

This thesis report presents performance analysis of wireless network over α-μ fading channel. We described the generalized nature of α-μ fading and revisited expressions for PDF, CDF, k^{th} moment, SNR, Outage Probability, MGF, BER, LCR, AFD, and Amount of Fading for α-μ fading channel. Using Monte-Carlo simulation in MATLAB, matching of analytical and simulated α-μ fading PDF are shown and outage & BER for different values of α and μ are discussed. We have analyzed the performance of block & cyclic coded a-μ fading channel with and without interleaving and compared with uncoded channel. We have found that on the cost of spectrum efficiency, coded link outperform uncoded link. Performance analysis of multi-user CDMA system with Walsh-Hadamard code is also presented for α-μ fading channel. It is observed that increase in number of users adversely affects the error performance, due to increase in multi-user interference.

Dual Diversity a-μ fading channel with SC, SSC, MRC and EGC combining is analysed for outage and BER performance. Further, correlated multi-antenna a-μ fading channel is also discussed with simulation results for different combining techniques. Analysis has been further extended and performance of SC, EGC and MRC combining, constant & exponential correlated multi-antenna a-μ fading channel are also investigated. It is seen that in multi-antenna receiver effected with correlated fading, performance of MRC is best among all and then the EGC and SC follows the performance subsequently. We also observed that the performance of wireless link effected with exponential correlation is superior than constant correlation.

We have shown the end-to-end performance of relay based communication system in AF and DF protocol for outage probability and bit error rate. In different path loss

condition, comparative performance study of AF & DF with direct link for two-hop relay-based system is presented with simulation results. It is established that in a regenerative relay-based communication, the optimal power allocated system outperforms the uniformly power allocated system. Outage and BER performance analysis for multi-hop serial relays have been presented for AF and DF protocols. Performance of cooperative communication of two-hop parallel multi-relays system has been analysed in AF and DF mode with MRC and SC. We have analysed the performance of HD and FD relay. It has been shown that with increase in parallel relays, performance deteriorates for HD system due to loss of spectrum efficiency but improves in FD system due to proper utilization of spectrum. It is established that MRC-DF outperforms in outage and BER with respect to MRC-AF and subsequently SC-DF and SC-AF, also the performance margin increases with increase in number of parallel relays.

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Abbreviations

AF	Amount of Fading, Amplify-and-Forward
AFD	Average Fade Duration
AOD	Average Outage Duration
AWGN	Additive White Gaussian Noise
BEP	Bit Error Probability
BER	Bit Error Rate
BPSK	Binary Phase-Shift Keying
CDF	Cumulative Distribution Function
CDMA	Code Division Multiple Access
DF	Decode-and-Forward
DL	Direct Link
DSSS	Direct Sequence Spread Spectrum
EDGE	Enhanced Data for GSM Evolution
EGC	Equal Gain Combining
FD	Full Duplex
GGD	Generalized Gamma Distribution
GPRS	General Packet Radio Service
GSM	Global System for Mobile
HD	Half Duplex
HDD	Hard Decision Decoding
HSDPA	High Speed Downlink Packet Access
HSUPA	High Speed Uplink Packet Access
LCR	Level Crossing Rate
LFSR	Linear Feedback Shift Register
LOS	Line-of-Sight
LT	Laplace Transform
LTE	Long Term Evolution
MATLAB	MATrix LABoratory
MGF	Moment Generating Function
MIMO	Multiple-Input Multiple-Output
MISO	Multiple-Input Single-Output

M-QAM	Multilevel Quadrature Amplitude Modulation
MRC	Maximal-Ratio Combining
MUI	Multi User Interferer/Interference
NLOS	Non Line-of-Sight
OFDM	Orthogonal Frequency Division Multiplexing
OP	Outage Probability
PDF	Probability Density Function
PN	Pseudo Noise
QPSK	Quadrature Phase-Shift Keying
RV	Random Variable
SC	Selection Combining
SDD	Soft Decision Decoding
SEP	Symbol Error Probability
SIMO	Single-Input Single-Output
SINR	Signal to (Interference + Noise) Ratio
SISO	Single-Input Single-Output
SNR	Signal to Noise Ratio
SSC	Switch and Stay Combining
UMTS	Universal Mobile Telecommunication Standard
WCDMA	Wideband Code Division Multiple Access
WiMAX	Worldwide Interoperability for Microwave Access

Notations/Symbols

α-μ	Alpha - mu
η-μ	Eta - mu
$f_X(x)$	Probability Density Function of Random Variable X
$F_X(x)$	Cumulative Distribution Function of Random Variable X
γ	Instantaneous fading SNR
$\overline{\gamma}$	Average fading SNR
$\Gamma(x)$	Gamma function [Gradshteyn and Ryzhik, 2007, Eq.(8.310.1)]
$\gamma(a,x)$	Lower Incomplete gamma function [Gradshteyn and Ryzhik, 2007, Eq.(8.350.1)]
$\Gamma(a,x)$	Complementary incomplete gamma function [Gradshteyn and Ryzhik, 2007, Eq.(8.350.2)]
i.i.d.	Independent and Identically Distributed
i.ni.d.	Independent and non-Identically Distributed
κ-μ	Kappa - mu
msv	Mean Square value
ρ	Correlation coefficient
$\hat{r}$	α-root mean value, $\hat{r} = \sqrt[\alpha]{E(r^{\alpha})}$
rms	Root Mean Square

CHAPTER 1: INTRODUCTION

1.1 Background:

Sir Jagdish Chandra Bose in November 1894 demonstrated using millimeter wavelength microwaves, to ignite gunpowder and rang a bell at a distance. Bose referred it *Adrisya Alok* (Invisible Light) and wrote "The invisible light can easily pass through brick walls, buildings etc. Therefore, messages can be transmitted by means of it without the mediation of wires." This was the first experience of wireless communication to the world, Bose (1899).

Later, Marconi in 1895, invented radio telegraph and wireless communication came into existence. Since then it is one of the most rapidly developing technologies and has generated huge interest for telecommunication engineers. It has come a long way from narrow-band voice communications to broadband multimedia communications to on-line high definition video calling and gaming application. The data rate of the wireless communications has increased dramatically, from kilobits per second to hundreds of megabits per second and expected to rise to Gigabits per second at the end of this decade.

In wireless communications, the radio signals may arrive at the receiver through line of sight (LOS) or non line of sight (NLOS) communication having multiple paths because of refection, diffraction, and scattering. This phenomenon is called multipath propagation, which causes constructive and destructive effect because of signal phase shifting. Channels with multipath fading fluctuate randomly, and therefore signal quality degrades significantly. When the bandwidth of the signal is greater than the coherence bandwidth of the fading channel, different frequency components of the signals experience different fading. This frequency-selective fading may further limit the data rate of wireless communications.

The statistics of mobile radio signal variation is described by Lognormal distribution for the long-term signal variation, and short-term signal variations is described by several distributions such as Rayleigh, Rician, Hoyt, Weibull, Nakagami-m etc. But it is important to mention that situations are encountered, where none of these distributions seems to adequately fit the experimental data. Therefore a general fading distribution namely, a-μ fading distribution, has been proposed by Yacoub (2002), which includes Nakagami-m, Weibull, Rayleigh etc. as special cases of a-μ distribution.

1.2 Motivation for Research:

From the beginning of 20th century, impressive developments in wireless technology have dramatically changed the way we communicate and live. This progress is continuing further as the demand for wireless connectivity to systems and devices is increasing day by day. Current wireless technology not only provides connectivity for voice and video data but also redefines the way in which we interact with technology. The physical wireless communication channel encounters many difficulties during transmission of wireless signals. While considering the design of wireless communication system, there are two major problematic factors which must be taken into account:

(i) Short term fading due to multipath propagation

(ii) Long term fading due to path loss factor

The wireless channels encounter deep fades and random fluctuations due to multipath propagation. The diversity schemes enhance reliability by introducing several copies of the same signal which are combined at the receiver using different combining techniques.

There are various diversity schemes, time diversity, frequency diversity, spatial diversity and the recent cooperative diversity. Cooperative diversity schemes can be seen as an extension of spatial diversity systems, where different relay nodes are placed in space as compared to a single multi-antenna source or receiver in conventional spatial diversity systems. Further, the cellular phones are too small to place multiple antennas in them, the multiple relays in cooperative systems provides solution to this issue. Other benefits are reduced requirement of high transmission power in a single node, reducing the interference level in the network, and achieving extended network coverage by deploying the relay nodes at end of cellular coverage areas.

Further, the a-μ fading distribution encompasses the both i.e. short-term and long-term propagation; therefore, performance study of cooperative diversity in a-μ fading distribution provides an adequate analysis for resolving the major issues of wireless channel in all the scenarios.

1.3 Overview of Generalized Fading:

The evolution of α-μ generalized distribution can be traced to discussion on multivariate Gamma-type distribution presented by Krishnamoorthy and Parthasarathy (1951) where n-variate distribution function representing a series have been provided. Later, generalization of Gamma distribution was reported by Stacy (1962) and physical basis for generalized Gamma distribution was given by Lienhard and Meyer (1967). The generalized Gamma distribution was renamed as "α-μ distribution" and its application for fading channels in wireless communication was first introduced by Yacoub (2002) with his landmark paper, "The α-μ distribution: A General Fading distribution". Further, Yacoub (2007a) exploring the nonlinearities of the propagation medium presented the statistics of α-μ distribution in closed form formula. In earlier works with help of indoor and outdoor field trial measurements carried out by Dias and Yacoub (2009), the autocorrelation and power spectrum functions of α-μ distribution have been derived and validated, whereas Selvati et.al. (2011) carried out field trials to obtain the probability density function and estimated α and μ fading parameters with their results. The accurate approximations for the outage probability of equal gain receivers subject to arbitrary independent co-channel interferers are proposed by Moraes (2008). The exact expressions for the level crossing rate and average fade duration has been derived by Da-costa (2007) for multi-branch selection, equal-gain, and maximal-ratio combiners operating over α-μ fading channels. Expressions for probability density function, cumulative distribution function for the product of two independent and non-identically distributed α-μ variates have been derived by Leonardo and Yacoub (2015). The joint probability density function and joint cumulative distribution function for multivariate α-μ distribution has been derived by De-Souza et. al. (2006, 2008). Based on moment estimators approach closed form approximations for the LCR of multi-branch equal-gain and maximal-ratio combiners operating on independent non-identically distributed Nakagami-m fading channels has been derived by Da-Costa and Yacoub (2007a). The exact expressions for LCR and AFD of equal-gain and maximal-ratio combiners have been derived by Da-Costa et. al. (2006). Alamouti (1998) described a simple transmit diversity technique for wireless communications. The average channel capacity for generalized fading scenarios are presented by Magableh and Matalgah (2011). Highly accurate closed-form approximations to PDF and CDF of the sum of i.i.d. α-μ variates have been provided by Da-Costa (2008). Moment generating function for the PDF characterizing the α-μ fading channel has been derived by Magableh and Matalgah (2009). MGF is further used for evaluating BER. Wang and Beaulieu (2009a), derived the expression for switching rate of a dual branch selection diversity combiner for α-μ and κ-μ generalized fading.

The Shannon capacity of the α-μ fading channel is derived by Mohamed et.al. (2012). Performance analysis of dual selection combining diversity receiver over correlated α-μ fading channels is presented by Panic et.al. (2009). Stefanovic et.al. (2009) provides performance analysis of signal-to-interference ratio based selection combining diversity system over α-μ fading distributed and correlated channels. Fading models for κ-μ distribution and η-μ distributions are presented by Yacoub (2007b). The performance of a dual-branch switched-and-stay combining diversity receiver, operating over correlated α-μ fading channel is discussed by Spalevic, et. al., (2010). The performance of the system with dual selection combining over correlated Weibull channel in the presence of α-μ distributed co-channel interference is studied by Petkovic, et. al., (2013). Capacity analysis of dual-hop wireless communication systems over α-μ fading channels is carried out by Magableh et.al. (2014). BER for i.i.d. α-μ fading channel with a maximal ratio combining receiver is carried out by Aldalgamouni et.al. (2013). Fraidenraich and Yacoub (2006) have discussed two new generalized fading distribution namely α-η-μ and α-κ-μ distributions. Performance analysis of wireless communication over α-η-μ fading channel has been investigated by Stamenovic et.al. (2014) and outage, PDF and CDF of received signal to interference ratio has been derived.

Khodabin and Ahmadabadi, (2010) have brought out that GGD is a flexible distribution and has exponential, gamma, and Weibull as subfamilies, and lognormal as limiting distribution. In flat fading environment channel estimation has been done by Sood et.al. (2010) using phase estimation of the transmitted signal to evaluate performance of OFDM-BPSK and -QPSK. A framework based on Mellin-transform for deriving closed form expression for symbol error rate of α-μ fading channel for single branch and maximal ratio combining receivers have been presented by Moataz et.al. (2014). Darawsheh and Jamoos (2014) discussed the problem of energy detection of an unknown deterministic signal over fading channel. Rabelo et.al. (2009) presented the κ-μ fading distribution, which is used for characterising the mobile radio propagation under severe fading conditions. Whereas Paris (2014) investigated natural generalization of the κ-μ fading channel in which the line of sight component is subject to shadowing. A novel characterisation of fading experienced in body to body communication channels is carried out by Cotton et.al. (2008) for fire and rescue personnel using the κ-μ distribution. Further, exact closed-form expression is derived for outage probability in η-μ fading channels by Jimenez and Paris (2010). Closed-form expressions for the averages of the Gaussian Q-function and product of two Gaussian Q-functions over the generalised η-μ and κ-μ distributions have been obtained by Ermolova (2009). BER performance of switched diversity receivers is analyzed by Pathak and Sahu (2014) over κ-μ and η-μ fading channels using moment generation function based approach. Performance analysis of α-η-μ fading channel is carried out by Panic et.al. (2013) when the communication is subjected to influence of co-channel interference.

Kostov (2003) presented MATLAB based approach for modelling of flat fading mobile radio channels. End-to-end performance of two-hops wireless communication systems with non regenerative relays over flat Rayleigh-fading channels is presented by Hasna and Alouni (2003). Various performance metrics of fading channels is discussed by Peppas et.al. (2011). Unified analytical framework is presented by Alouini and Goldsmith (1999) to determine the exact average symbol-error rate of linearly modulated signals over generalized fading channels. Unified approach for evaluating the error rate performance of digital communication systems operating over a generalized fading channel is given by Simon and Alouini (1998). Yilmaz and Alouini (2010) developed a novel generic framework for the capacity analysis of L-branch equal gain combining/maximal ratio combining over generalized fading channels. Kaur (2013) explained how cognitive wireless networks fulfills the need of additional wireless spectrum. Deployment of multi-input multi-output antenna systems with orthogonal frequency division multiplexing over wireless channels has been identified by Oluwafemi and Mneney (2013) as one of the most promising techniques for future wireless services. Liang and Veeravalli (2007) presented cooperative relay broadcast channels. Ameen and Yousif (2014) described decode and forward cooperative protocol enhancement using interference cancellation.

1.4 Research Objective:

In past one decade extensive research has been done on generalized a-μ fading channels, because of their applicability in various fading models. The objective of this thesis report is to analyse the performance of wireless network over a-μ fading channel. In general, the thesis looks at the performance evaluation of different wireless system models over a-μ fading channel in order to improve their outage probability and bit error rate performance.

Cooperative communication is also one of the extremely fertile research area in wireless in recent past. During our literature survey we have not come across any research paper on performance analysis of multi-antenna or relay-based communication systems over a-μ fading channel. We propose to carry out performance analysis of different wireless communication systems over a-μ fading channel and finally analyze the cooperative communication over a-μ fading channel.

1.5 Thesis Organization:

The thesis report is organised as follows. The chapter 1, begins with brief introduction, motivation objective of the thesis report. Overview of extensive literature survey on the previous work carried out in this field has been summarized. Chapter 2 describes multipath fading, parameters of fading channels and classification of fading channels. Short-term fading channel models such as Rayleigh, Rician, Nakagami-*q* (Hoyt), Nakagami-*n* (Rice), Nakagami-*m*, and Weibull have been explained. Multipath diversity methods and wireless system performance metrics such as average SNR, outage probability, average bit error rate (BER), amount of fading, average fade duration and level crossing rates have been defined.

The chapter 3, begins with generalized fading channels a-μ, η-μ, κ-μ, and explains the generalized nature of a-μ. The mathematical formula of a-μ fading has been derived. The closed form expression for probability density function, cumulative distribution function, the k^{th} moment, signal to noise ratio, outage probability, moment generating function, bit error rate, level crossing rate, average fade duration, and amount of fading have been derived which helps in characterisation and performance analysis of a-μ fading.

In chapter 4, performance analysis of a-μ fading channel has been carried out by Monte-Carlo simulation in MATLAB. Plots for probability density function, outage, and BER for various values of a and μ parameters are shown. Further, block and cyclic codes, interleaving techniques such as random, block and convolution is explained. Performance analysis of block & cyclic coded a-μ fading channel with and without interleaving is compared with uncoded channel. Performance analysis of multi-user CDMA system with Walsh-Hadamard code over a-μ fading channel is also performed.

The chapter 5, extends the work of chapter 4 for multi-antenna a-μ fading channel. Combining techniques such as selection combining (SC), switch and stay combining

(SSC), maximal ratio combining (MRC), and equal gain combining (EGC) have been described and outage and BER performance over a-μ fading channel have been simulated and result is discussed. Fading correlation in correlated multi-antenna a-μ fading channel is stated, and simulated comparative performance with various combining techniques such as SSC, SC, EGC and MRC is brought out. Analysis has been further extended and performance of SC, EGC and MRC combining, coded, constant & exponential correlated multi-antenna a-μ fading channel is also investigated.

In chapter 6, end-to-end performance of relay based communication system in amplify-and-forward (AF) protocol and decode-and-forward (DF) protocol have been analysed for outage probability and bit error rate. Path loss in radio propagation model is briefly discussed. In different path loss exponent condition, comparative performance study of AF & DF with direct link (DL) for two-hop relay-based system is presented with simulation results. It has been also shown with simulation results that in a regenerative relay-based communication, the optimally power allocated system out performs the uniformly power allocated system. Outage and BER performance analysis for multi-hop serial relays has been presented for AF and DF protocols. Performance of Cooperative communication of two-hop parallel multi-relays system have been analysed in AF and DF mode with maximal ratio combining and selection combining. The chapter 7, concludes thesis report and discusses some further research areas which can be explored in future.

CHAPTER 2

WIRELESS CHANNEL

2.1 Multipath Fading:

In wireless channel multiple propagation paths exist from transmitter to receiver due to reflection, diffraction, and scattering of wave by different objects. Signal copies received through different path can undergo rapid attenuation, distortion, delays and phase shift over a short period of time due to constructive or destructive interference occurring at receiver. In destructive interference signal power decreases significantly and this phenomenon is called as fading. The multiple propagation path are time-varying. This results in multipath time-varying fading channel. The time-varying nature of channel is the major constraint for reliable wireless transmission. Multipath fading results in severe performance degradation in terms of error probability and poor reliability of wireless communication system.

Fading occurring over a short period of time or small travel distance is called small-scale fading. As we consider only smaller distances hence we can ignore large scale path loss. Multipath propagation, speed of mobile, speed of surrounding objects, and the transmission bandwidth of the signal are the important factors influencing small-scale fading

2.1.1 Parameters of Fading Channels:

Following terms and parameters (Rappaport, 2011) are often used to characterize and quantify different multipath channels:

(i) Delay Spread: This is time dispersive property of a channel, for multi-path signals having significant power.

$$\text{Mean Excess Delay, } \bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)(\tau_k)}{\sum_k P(\tau_k)} \tag{2.1}$$

$$\text{RMS Delay Spread, } \quad \sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2} \tag{2.2}$$

$$\text{where } \overline{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)(\tau_k^2)}{\sum_k P(\tau_k)}$$

where a_k is the amplitude and $P(\tau_k)$ is the power associated with k^{th} multipath component arriving at τ_k time instant.

(ii) Coherence Bandwidth (B_C): Range of frequencies over which the channel is considered as flat (i.e. channel passes all spectral components with equal gain and linear phase).

(iii) Doppler Spread (B_D): It gives maximum range of Doppler shift. ($B_D = v / \lambda$)

(iv) Coherence Time (T_C): is the time duration over which the channel impulse response is essentially invariant.

$$T_C \approx \sqrt{\frac{9}{16\pi f_m^2}} = \frac{0.423}{f_m} \tag{2.3}$$

2.1.2 Classification of Fading Channels:

Depending on the parameters of the channels and the characteristics of the signal to be transmitted, time-varying fading channels can be classified as:

(i) Flat fading versus frequency selective fading: This classification is made on the basis of time delay spread. If BW of transmitted signal (B_S) is small compared to coherence bandwidth (B_C), then all frequency component of signal would roughly undergo same amount of fading. Then the channel is classified as *flat fading*. In this rms delay spread (σ_τ) is less than symbol period (T_S). On the other hand, if BW B_S is large compared to B_C, then different frequency component of signal would undergo different degree of fading. Then the channel is classified as *frequency selective fading*. In this rms delay spread (σ_τ) is greater than symbol period (T_S).

(ii) Fast fading versus slow fading: This classification is made on the basis of Doppler spread. If BW of transmitted signal (B_S) is small compared to Doppler spread (B_D), then channel impulse response changes rapidly within the symbol duration. Then the channel is classified as *fast fading*. In this symbol period (T_S) is greater than coherence time (T_C). On the other hand if BW of transmitted signal (B_S) is large compared to Doppler spread (B_D), then channel impulse response changes at a rate much slower than the transmitted base band signal. Then the channel is classified as *slow fading*. In this symbol period (T_S) is less than coherence time (T_C).

The above classification of fading channels depends on the properties of transmitted signal. The two way classification based on time delay spread and Doppler spread gives rise to four different types of channel:

- Flat Slow fading
- Flat Fast fading
- Frequency Selective Slow fading
- Frequency Selective Fast fading

2.2 Fading Channels:

The radio propagation environment nature describes the statistical behavior of radio signals. There are various distributions given in Simon and Alouini (2005), which describe the statistics of the radio mobile signals. Long-term signal variations are characterized by lognormal distribution. The short-term distribution is described by several models such as

Rayleigh, Ricean, Nakagami-*n* (Rice), Nakagami-*m*, Nakagami-*q* (Hoyt), and Weibull distribution.

2.2.1 Rayleigh Fading: The Rayleigh distribution is frequently used to model multipath fading with no direct line-of sight (LOS) path between transmitter and receiver antennas. The received envelope of a flat fading signal or the envelope of an individual multipath component is given by Rayleigh distribution with PDF of channel fading amplitude α as

$$f_\alpha(\alpha) = \frac{\alpha}{\sigma^2} e^{\left(\frac{-\alpha^2}{2\sigma^2}\right)}; \qquad \alpha \geq 0 \tag{2.4}$$

Where σ is the standard deviation and σ^2 is the variance of the received signal before detection. Rayleigh Fading exists in urban area where there is no line-of-sight (LOS) communication.

$$\text{Instantaneous SNR, } \gamma = \frac{\text{Signal power}}{\text{Noise power}} = \frac{\alpha^2 E_s}{N_0} \tag{2.5}$$

where E_s is energy per symbol. Therefore,

$$\text{Average SNR, } \bar{\gamma} = \frac{\overline{\alpha^2}\ \mathrm{E_s}}{\mathrm{N_0}} = \frac{\sigma^2 E_s}{N_0} \tag{2.6}$$

$$\text{Also, } \bar{\gamma} = \int_0^\infty \gamma\ \mathrm{p}_\gamma(\gamma)\ \mathrm{d}\gamma \tag{2.7}$$

$$\text{From above, } \frac{\gamma}{\bar{\gamma}} = \frac{\alpha^2}{\sigma^2} \tag{2.8}$$

$$\text{or, } \alpha = \sqrt{\frac{\gamma\ \sigma^2}{\bar{\gamma}}} \tag{2.9}$$

Therefore, PDF of γ can be obtained by change of variable method as given in Appendix –A6.

Thus, PDF of γ may be given as

$$f_\gamma(\gamma) = f_\alpha\left(\sqrt{\frac{\gamma\ \sigma^2}{\bar{\gamma}}}\right) \times \frac{d}{d\gamma}\left(\sqrt{\frac{\gamma\ \sigma^2}{\bar{\gamma}}}\right) \tag{2.10}$$

$$f_\gamma(\gamma) = \frac{2}{\sigma^2}\left(\sqrt{\frac{\gamma\ \sigma^2}{\bar{\gamma}}}\right) \exp\left(-\frac{\frac{\gamma\ \sigma^2}{\bar{\gamma}}}{\sigma^2}\right) \times \left(\sqrt{\frac{\sigma^2}{\bar{\gamma}}}\right)\frac{1}{2}\gamma^{-\frac{1}{2}} \tag{2.11}$$

on sloving we get, $f_\gamma(\gamma) = \frac{1}{\bar{\gamma}}\ \mathrm{e}^{-\frac{\gamma}{\bar{\gamma}}}$; for $\gamma \geq 0$ (2.12)

Thus, PDF of instantaneous SNR (γ) for Rayleigh fading is given as

$$f_\gamma(\gamma) = \frac{1}{\bar{\gamma}}\ e^{\left(-\frac{\gamma}{\bar{\gamma}}\right)}\ ;\quad for\ \gamma \geq 0 \tag{2.13}$$

Moment Generating Function (MGF) of Rayleigh Fading is

$$\mathrm{M}_\gamma(s) = \int_0^\infty f_\gamma(\gamma)\ e^{s\gamma}\ d\gamma \tag{2.14}$$

This is same as Laplace Transform (LT) formula with (-s) replaced with (s), and (t) with (γ). i.e.

$$L.T.\ of\ [f_\gamma(\gamma)] = P_\gamma(s) = \mathrm{M}_\gamma(-s) \tag{2.15}$$

Therefore, $\mathrm{M}_\gamma(s) = L.T.\ of\ [f_\gamma(\gamma)]_{s=-s}$ (2.16)

or, $\mathrm{M}_\gamma(s) = L.T.\ of\ \left[\frac{1}{\bar{\gamma}}\ e^{\left(-\frac{\gamma}{\bar{\gamma}}\right)}\right]_{s=-s}$ (2.17)

or, $\mathrm{M}_\gamma(s) = \left[\frac{1}{\bar{\gamma}}\frac{1}{(-s+\frac{1}{\bar{\gamma}})}\right]$ (2.18)

or, $\mathrm{M}_\gamma(s) = \left[\frac{1}{(1-s\bar{\gamma})}\right]$ (2.19)

Thus, MGF of Rayleigh Fading is given as

$$\mathrm{M}_{\gamma}(s) = \left(1 - s\overline{\gamma}\right)^{-1} \tag{2.20}$$

2.2.2 Rician:

The small scale fading distribution is Rician, when a dominant stationary i.e. non-fading signal component such as a LOS propagation path is present. The random multipath components are superimposed on a stationary dominant signal. The effect of the dominant signal arriving with many weaker multipath signals gives rise to the Rician distribution. The Rician distribution becomes Rayleigh distribution when the dominant component fades away. The PDF of Rician fading envelope is given as

$$f_{\alpha}(\alpha) = \frac{\alpha}{\sigma^2} e^{\left[\frac{-(\alpha^2 + A^2)}{2\sigma^2}\right]} I_0\left(\frac{A\alpha}{\sigma^2}\right) ; \quad \mathit{for}\ (A \geq 0,\ \alpha \geq 0) \tag{2.21}$$

Where the parameter *A* denotes the peak amplitude of the dominant signal, and $I_0(*)$ is the modified Bessel function of the first kind and zero order.

2.2.3 Nakagami-*q* (Hoyt):

The Nakagami-*q* fading envelope, also referred as *Hoyt* fading envelope is given as

$$f_{\alpha}(\alpha) = \frac{(1+q^2)\alpha}{q\,\Omega} e^{\left(-\frac{(1+q^2)^2\alpha^2}{4q^2\,\Omega}\right)} I_0\left(\frac{\left(1-q^4\right)\alpha^2}{4q^2\Omega}\right); \qquad \alpha \geq 0 \tag{2.22}$$

Where $I_0(.)$ is the zeroth-order modified Bessel function of the first kind and q is the Nakagami-*q* fading parameter, which ranges from 0 to 1. The Nakagami-*q* spans the range from one sided Gaussian fading (q = 0) to Rayleigh fading (q = 1). It is generally observed on satellite links subjects to strong ionospheric scintillation. It is important to note that one-sided Gaussian fading corresponds to the worst case of fading, and has largest Amount of Fading.

2.2.4 Nakagami-*n* (Rice):

The Nakagami-*n* fading envelope is also referred as *Rice* fading envelope. It is used to model propagation paths consisting of one strong direct LOS component and many random weaker components. In this the channel fading amplitude α follows the distribution as

$$f_\alpha(\alpha) = \frac{2(1+n^2)e^{-n^2}\alpha}{\Omega} e^{\left(-\frac{(1+n^2)^2\alpha^2}{\Omega}\right)} I_0\left(2n\alpha\sqrt{\frac{1+n^2}{\Omega}}\right); \qquad \alpha \geq 0 \tag{2.23}$$

Where *n* is the Nakagami-*n* fading parameter, which ranges from 0 to ∞. The Nakagami-*n* spans the range from Rayleigh fading (*n* = 0) to no fading i.e. constant amplitude (*n* = ∞). It is typically observed in the first resolvable LOS paths of microcellular urban and sub-urban land-mobile, pico-cellular indoor and factory environments. It also appears to the dominant LOS path of satellite and ship to ship radio links.

2.2.5 Nakagami-*m*:

The Nakagami-m PDF of fading envelope, is basically a central chi-square distribution. Here the channel fading amplitude *α* follows the distribution as

$$f_\alpha(\alpha) = \frac{2\ \mathrm{m}^m\ \alpha^{2m-1}}{\Omega^m\ \Gamma(m)}\ e^{\left(-\frac{m\alpha^2}{\Omega}\right)}; \qquad \alpha \geq 0 \tag{2.24}$$

Where *m* is the Nakagami-*m* fading parameter, which ranges from ½ to ∞. The Nakagami-*m* spans a very wide range from one sided Gaussian distribution (*m* = ½) and Rayleigh fading (*m* = 1) to non fading AWGN channel (*m* = ∞). Nakagami-*m* fading distribution often gives the best fit to land-mobile and indoor-mobile multipath propagation, as well as scintillating ionospheric radio links.

2.2.6 Weibull:

The Weibull fading envelope is another mathematical description of a probability model for characterizing amplitude fading in a multipath environment, particularly that associated with mobile radio systems operating in the 800/900 MHz frequency range. The Weibull PDF is given by

$$f_{\alpha}(\alpha) = c\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{\Omega}\right)^{\frac{c}{2}} \alpha^{c-1} \ \mathrm{e}^{\left(-\left(\frac{\alpha^2}{\Omega}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}\right)}; \qquad \alpha \geq 0 \tag{2.25}$$

Where c is a parameter that is chosen to yield a best fit to measurement results and as such affords the shape flexibility of the Nakagami distributions.

2.3 Multipath Diversity Methods:

Diversity is a very effective communication technique that provides higher data rate, mitigates effect of fading, and brings effective improvement in wireless link at relatively low cost. In diversity, the receiver is provided with multiple copies of the same message, which helps effectively in combating channel impairment occurring due to multipath fading.

2.3.1 Space Diversity: Space diversity is a method of transmission or reception, or both, in which the effects of fading are minimized by the simultaneous use of two or more physically separated antennas. Thus, space diversity requires multi-antennas which can be used at either mobile or base station, or both. Spatially separated antennas on the devices with separations of one half wavelength or more will have uncorrelated envelopes. Base station antennas must be separated far apart on the order of several tens of wavelength to achieve de-correlation.

Multi-antenna wireless communication systems are shown in Fig.2.1.

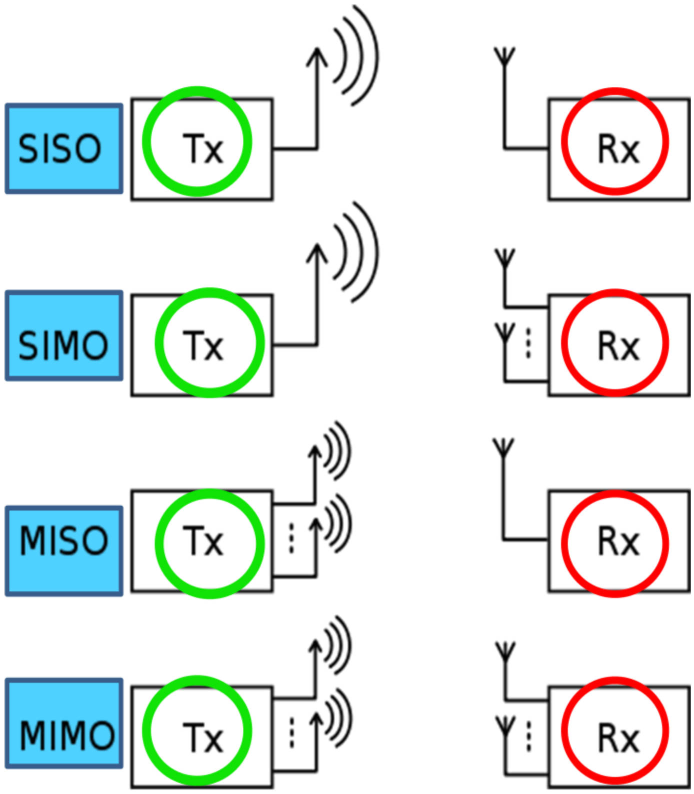

Fig.2.1. Multi-antenna Wireless Communication systems.

(i) Single Input Single Output (SISO) Systems: This is the simplest communication link between one transmit antenna and one receive antenna. Therefore in this spatial diversity can not be applied.

(ii) Single Input Multiple Output (SIMO) Systems: This communication systems has one transmit antenna and multiple antennas at receiver side. The transmitted signal from single transmit-antenna arrives at all receiver antennas through different channels and it is assumed that channels are completely de-correlated. Thus multiple independent copies of same signal arrive at receiver. With help of any of the combining technique such as selection, maximal ratio or equal gain combining at receiver, SNR at output can be maximized.

(iii) Multiple Input Single Output (MISO) Systems: MISO communication systems uses multiple antennas at the transmitter and a single antenna at the receiver.

(iv) Multiple Input Multiple Output (MIMO) Systems: MIMO communication systems uses multiple antennas at both the transmitter and receiver. In MIMO systems generally space-time diversity is employed. The Alamouti space time block code is the simplest code belonging to the family of orthogonal space time code.

2.3.2 Frequency Diversity: In frequency diversity the same information signal is transmitted and received simultaneously on two or more independent fading carrier frequencies. The idea behind this technique is that frequencies separated by more than the coherence bandwidth of the channel will not experience the same fading. This is employed in microwave line-of-sight links which carry several channels in a frequency division multiplex mode (FDM).

2.3.3 Time Diversity: The signal representing the same information are sent over the same channel at different times.. Time diversity is obtained by repeatedly transmitting same information at time spacing that are separated by at least the coherence time of the channel. Thus multiple repetitions of the signal is received with independent fading conditions, thereby providing diversity, an example is RAKE receiver.

2.4 System Performance Measures:

2.4.1 Average SNR:

Signal to Noise Ratio (SNR) is the most common and very well understood performance measure characteristics of a digital communication. Generally, it is measured at output of receiver thus directly related to data detection process. Among all performance measure parameters it is easiest to evaluate, and indicates overall system performance. The term average refers to statistical averaging over the probability distribution of the fading.

$$\text{Average SNR,} \quad \bar{\gamma} = \int_0^{\infty} \gamma \, f_{\gamma}(\gamma) \; \mathrm{d}\gamma \tag{2.26}$$

Where, γ = instantaneous SNR and is given as

$\gamma = \sum_{l=1}^{L} \gamma_l$ where L denotes the number of channel combined.

2.4.2 Outage Probability (P_{out}):

P_{out} is the probability that the output SNR (γ), falls below a a certain specified threshold (γ_{th}).

$$P_{out} = \int_0^{\gamma_{th}} f_{\gamma}(\gamma) \; \mathrm{d}\gamma \tag{2.27}$$

P_{out} is also Cumulative Distribution Function (CDF).

$$\text{PDF and CDF are related as,} \quad f_{\gamma}(\gamma) = \frac{d}{d\gamma} \, \mathrm{F}_{\gamma}(\gamma) \tag{2.28}$$

2.4.3 Average Bit Error Rate (BER):

Bit Error Probability (BEP) = Bit Error Rate (BER)

Symbol Error Probability (SEP) = Symbol Error Rate (SER)

BER is difficult to compute, but it is the most revealing about the nature of the system behavior. It is extensively used in documents containing system performance evaluations.

$$BER = \int_0^{\infty} \mathrm{Q}\left(a\sqrt{\gamma}\right) f_{\lambda}(\gamma) \; \mathrm{d}\gamma \tag{2.29}$$

Where Q is Gaussian Q-function and a is constant given by eq. (5.1) of Simon and Alouni (2005) and depends on modulation and detection combination. BER evaluation is difficult because it is a non-linear function of γ. However, we find that an MGF based approach is still quite useful in simplifying the analysis.

2.4.4 Amount of Fading (AF):

For a fading channel, the amount of fading (AF) given by eq. (2.30) is computed at the output of combiner and reflects the behavior of the particular diversity combining technique. AF is defined by

$$AF = \frac{\text{var}\ \gamma_t}{(E[\gamma_t])^2} = \frac{E(\gamma_t^2) - (E[\gamma_t])^2}{(E[\gamma_t])^2} \tag{2.30}$$

where, γ_t = total instantaneous SNR, $E(\gamma_t^2) =$ mean square value,
$E[.] =$ statistical average or mean, $\text{var}[.] =$ variance

2.4.5 Average Fade Duration (AFD):

AFD is also called as Average Outage Duration (AOD).

AFD = $T(\gamma_{th})$ = is the measure of how many (in seconds), on the average the system remains in the outage stage.

$$AFD = T(\gamma_{th}) = \frac{P_{out}}{N(\gamma_{th})} \tag{2.31}$$

where $\text{N}(\gamma_{th}) =$ is frequency of outage = average Level Crossing Rate (LCR)

2.4.6 Level Crossing Rate (LCR):

LCR is defined as the expected rate at which the fading envelope, normalized to the local *rms* signal level, crosses a specified level in a positive going direction. For Rayleigh's fading LCR is given as

$$N_R = \int_0^\infty \dot{r}\ \rho(R,\dot{r})\ d\dot{r} = \sqrt{2\pi}\ f_m\ \rho\ e^{-\rho^2} \tag{2.32}$$

CHAPTER 3

ALPHA-MU (α-μ) FADING

3.1 GENERALIZED FADING CHANNELS:

3.1.1 Alpha-mu (α-μ):

The Alpha-mu (α-μ) fading channel model initially discussed by Yacoub (2002) in his landmark paper stating that, the Alpha-mu (α-μ) is a general fading distribution used to represent the small scale variation of the fading signal.

For a fading signal with envelope r, and an arbitrary parameter $\alpha > 0$,

and α-root mean value, $\hat{r} = \sqrt[\alpha]{E(r^{\alpha})}$

the α-μ probability density function, $f_R(r)$ of r is given as

$$f_R(r) = \frac{\alpha\ \mu^{\mu}\ \mathrm{r}^{\alpha\mu - 1}}{\hat{r}^{\alpha\mu}\ \Gamma(\mu)} \quad \exp\left[-\mu \frac{r^{\alpha}}{\hat{r}^{\alpha}}\right] \tag{3.1}$$

Where $\mu \geq 0$ is the inverse of the normalized variance.

$\Gamma(z) = \int_0^{\infty} t^{z-1} \mathrm{e}^{-t} \mathrm{dt}$ is the Gamma function.

The *(α-μ)* is a general distribution which includes the Weibull and the Nakagami-m distributions as special cases. Weibull includes the Rayleigh and the Negative Exponential distributions whereas Nakagami-m includes the Rayleigh and the One-Sided Gaussian distributions. The Lognormal distribution may also be well approximated by the *(α-μ)* Distribution.

3.1.2 Eta-mu (η-μ):

Yacoub (2007b), has described that eta-mu (η-μ) is a general fading distribution used to represent the small scale variation of the fading signal in non-LOS condition. This distribution appears in two different formats. For a fading signal with envelope, R, and normalized envelope, ρ = R/R$_{rms}$, the η-μ envelope probability density function, $f_\rho(\rho)$,for both formats is given in unified way as

$$f_\rho(\rho) = \frac{4\sqrt{\pi}\ \mu^{\mu+\frac{1}{2}} h^{\mu}}{\Gamma(\mu)\ H^{\mu-\frac{1}{2}}}\ \rho^{2\mu}\ e^{(-2\mu h \rho^2)}\ \times I_{\mu-\frac{1}{2}}\left[2\mu H \rho^2\right] \tag{3.2}$$

Where *h, H* are function of η and are defined for both formats as below**.**

In *format 1*, ($0 < \eta < \infty$) is the scattered-wave power ratio between the in-phase and quadrature components of each cluster of multipath. The in-phase and quadrature components of the fading signal within each cluster are assumed to be independent from each other and to have different powers.

$$h = \frac{2 + \eta^{-1} + \eta}{4}\ ,\quad H = \frac{\eta^{-1} - \eta}{4}\ ,\ and\ \frac{H}{h} = \frac{1-\eta}{1+\eta} \tag{3.3}$$

In *format 2*, ($-1 < \eta < 1$) is the correlation coefficient between the scattered-wave in-phase and quadrature components of each cluster of multipath. The in-phase and quadrature components of the fading signal within each cluster are assumed to have identical powers and to be correlated with each other.

$$h = \frac{1}{1 - \eta^2}\ ,\quad H = \frac{\eta}{1 - \eta^2}\ ,\ and\ \frac{H}{h} = \eta \tag{3.4}$$

The *(η-μ)* is a general distribution that includes the Hoyt (Nakagami-*q*), the One-Sided Gaussian, the Rayleigh, and, more generally, the Nakagami-*m* distributions as special cases.

3.1.3 Kappa-mu (*κ-μ*):

The Kappa-mu (***κ-μ***) discussed by Yacoub (2007b), is a general fading distribution used to represent the small scale variation of the fading signal in LOS condition. For a fading signal with envelope R, and normalized envelope, ρ = R/R$_{rms}$, the κ-μ envelope probability density function, $f_\rho(\rho)$,is given as

$$f_\rho(\rho) = \frac{2\ \mu\ (1+\kappa)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu\text{-}1}{2}}\ \mathrm{e}^{\mu\kappa}}\ \rho^{\mu}\ \mathrm{e}^{\left[-\mu(1+\kappa)\rho^2\right]} \times I_{\mu-1}\left[2\mu\sqrt{\kappa(1+\kappa)\rho}\ \right] \tag{3.5}$$

Where $\kappa > 0$ is the ratio between the total power of the dominant components and the total power of the scattered waves, $\mu > 0$, and $I_v\,(.)$ is the modified Bessel function of the first kind and order *v*. The fading model for the **(**κ-μ) distribution considers a signal composed of clusters of multipath waves, propagating in a non-homogeneous environment. The *(*κ-μ*)* is a general distribution which includes the Rice and Nakagami-m distributions. Both the Rice and Nakagami-*m* distributions include the Rayleigh distribution, and, the Nakagami-m distribution includes the One-Sided Gaussian distribution. Therefore, these distributions can also be obtained from the (κ-μ) distribution.

3.2 *α-μ* : GENERALIZED FADING

The α-μ is a general fading distribution, that can be used to represent various fading models. The α-μ distribution includes Nakagami-*m* and Weibull as special cases. Further, Nakagami-*m* includes One-Sided Gaussian and Rayleigh distributions, whereas Weibull includes Negative Exponential and Rayleigh distributions. The Lognormal distribution may also be approximated by α-μ distribution. Thus α-μ distribution is a generalized fading distribution. The α-μ probability density function, $f_R(r)$ of r is given as

$$f_R(r) = \frac{\alpha\ \mu^{\mu}\ \mathrm{r}^{\alpha\mu\ \text{-}\ 1}}{\hat{r}^{\alpha\mu}\ \Gamma(\mu)}\quad \exp\left[-\mu\frac{r^{\alpha}}{\hat{r}^{\alpha}}\right] \tag{3.6}$$

3.2.1 Weibull distribution: The Weibull distribution is a probability model for characterizing amplitude fading in a multipath environment, associated with mobile radio systems operating in the 800/900 MHz frequency range. Weibull distribution is obtained from α-μ distribution by setting μ =1, in eq. (3.6).

$$f_R(r) = \alpha\ \beta\ \ \mathrm{r}^{\alpha\ \text{-}\ 1}\quad \exp\left[-\beta\ r^{\alpha}\right], \qquad \textit{where}\ \beta = \hat{\mathrm{r}}^{\text{-}\alpha} \tag{3.7}$$

3.2.2 Rayleigh distribution: Rayleigh distribution models multipath fading for mobile systems where no line-of-sight (LOS) path exists between the transmitter and receiver antennas. From the Weibull distribution, by setting α =2 in eq. (3.6), the Rayleigh distribution can be obtained.

$$f_R(r) = \frac{r}{\gamma^2} \quad \exp\left[-\frac{r^2}{2\gamma^2}\right], \quad where\ \gamma^2 = \frac{\hat{r}^2}{2} \tag{3.8}$$

3.2.3 Negative Exponential distribution: From Weibull distribution, negative exponential distribution can also be obtained by setting α =1 in eq. (3.6).

$$f_R(r) = \delta \ \exp\left[-\delta\, r\right], \quad \text{where } \delta = \hat{\mathrm{r}}^{-1} \tag{3.9}$$

3.2.4 Nakagami-m distribution: The Nakagami-*m* probability density function is in essence a central chi-square distribution. The Nakagami-*m* can be obtained from α-μ distribution PDF by setting α =2 in eq. (3.6),

$$f_R(r) = \frac{2\ \mu^{\mu}\ \mathrm{r}^{2\mu - 1}}{\Omega^{\mu}\ \Gamma(\mu)} \quad \exp\left[-\mu\frac{r^2}{\Omega}\right], \quad \text{where } \Omega = \hat{\mathrm{r}}^2 \tag{3.10}$$

3.2.5 Rayleigh distribution: From Nakagami-*m*, by setting μ =1, Rayleigh distribution can again be obtained.

$$f_R(r) = \frac{r}{\gamma^2} \quad \exp\left[-\frac{r^2}{2\gamma^2}\right], \quad \text{where } \gamma^2 = \frac{\hat{\mathrm{r}}^2}{2} \tag{3.11}$$

3.2.6 One-sided Gaussian distribution: From Nakagami-*m*, One-sided Gaussian distribution is also obtained by setting μ = ½ .

$$f_R(r) = \sqrt{\frac{2}{\pi\ \hat{r}^2}} \quad \exp\left[-\frac{r^2}{2\ \hat{r}^2}\right] \tag{3.12}$$

Various fading distributions for different value of α and μ are given in Table 3.1:

Table 3.1: Fading distributions for different values of α and μ

Fading Distribution	***α* - value**	***μ* - value**
Exponential	1	1
Weibull (between Rayleigh and Exponential)	1.5	1
Rayleigh	2	1
Nakagami-m (one-sided Gaussian $\mu=½$)	2	0.5
Nakagami-m (Chi $\mu=5$)	2	5
Gamma (Chi-square $\alpha=5$)	5	1

3.3 FUNDAMENTALS OF *α-μ* FADING:

Wireless communication system in multipath fading channel as shown in fig. 3.1, depicts the trees and buildings as clusters in radio propagation and shows the local scatters near the receiver. In the recent past alpha-mu (α-μ) fading model has been proposed considering two important phenomenon of radio propagation non-linearity and clustering.

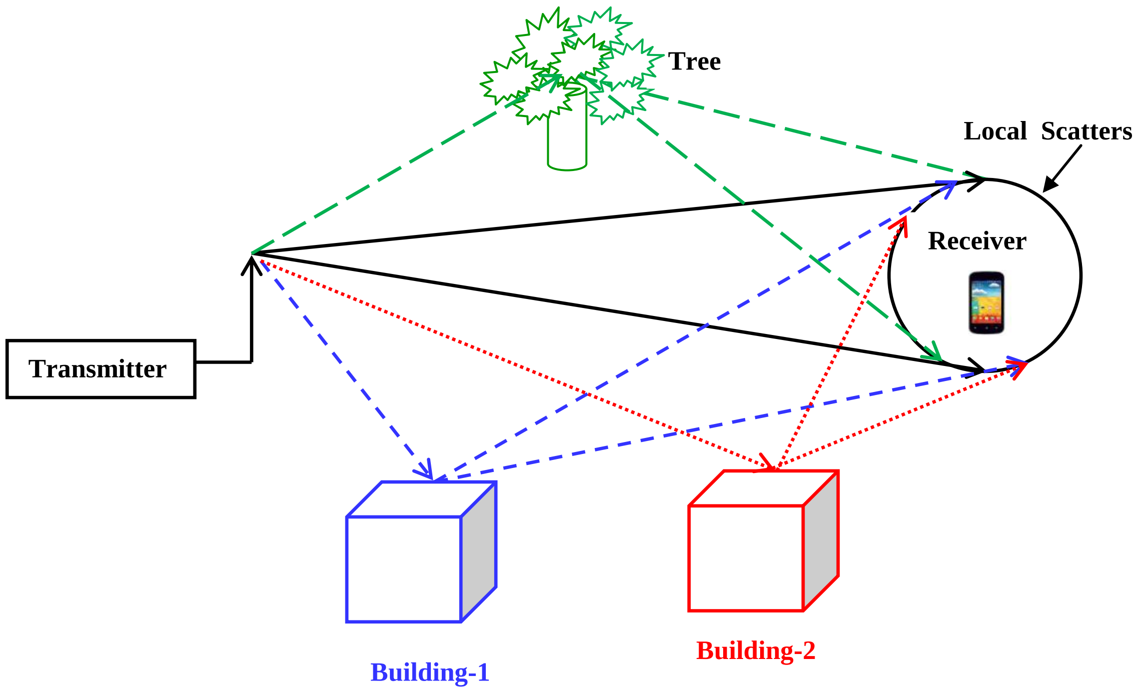

Fig.3.1. Wireless Communication System in Multipath Fading Channel

The α-μ represents a generalized fading distribution for small-scale variation of the fading signal in a non line-of-sight fading condition. In fact, the Generalized Gamma Distribution (GGD) also known as Stacy distribution has been renamed as α-μ distribution, which indicates the physical parameters involved. The α-μ distribution is flexible, general and has easy mathematical tractability. As given in its name, alpha-mu distribution is written in terms of two physical parameters, namely α and μ. The power parameter ($\alpha > 0$) is related to the non-linearity of the environment i.e. propagation medium, whereas the parameter ($\mu > 0$) is associated to the number of multipath clusters. In the α-μ distribution, it is considered that a signal is composed of clusters of multipath waves propagating in a non-homogenous environment. In any one of the cluster, the phases of the scattered waves are random and have similar delay times. Further, the delay-time spreads of different clusters is generally relatively large. It is assumed that the clusters of multipath waves have the scattered waves with identical powers. As a result, the obtained envelope, is a non-linear function of the modulus of the sum of the multipath components. The non-linearity is given in terms of propagation medium magnitude parameter ($\alpha > 0$) and multipath clustering magnitude parameter ($\mu > 0$).

3.3.1 Probability Density Function (PDF) of α-μ:

Assuming that the received signal has n multipath components, and non-linearity of propagation environment is in the form of power α, the resulting α-μ envelope observed as modulus of sum of multipath components with power $2/\alpha$. Hence the envelope r as function of in-phase and quadrature elements will be

$$r = \left[\sum_{i=1}^{n} \left(x_i^2 + y_i^2 \right) \right]^{\frac{1}{\alpha}} \tag{3.13}$$

x_i and y_i are mutually independent Gaussian processes with mean $\mathrm{E}(x_i) = E(y_i) = 0$

$$\text{and variance } V(x_i) = V(y_i) = \sigma^2 \tag{3.14}$$

$$\hat{r} = \sqrt[\alpha]{E(r^\alpha)} = \sqrt[\alpha]{2\sigma^2 n} = \sqrt[\alpha]{2\sigma^2 \mu} \tag{3.15}$$

with E(.) and $V(\cdot)$ denoting mean and variance operators respectively.

Let us define $y = r^\alpha$, hence following standard procedure we find

$$f_Y(y) = \frac{y^{n-1}}{(2\sigma^2)^n\ \Gamma(n)} \exp\left(-\frac{y}{2\sigma^2}\right), \tag{3.16}$$

By using transformation of variables defined in Appendix –A6, from eq. (3.16) we get

$$f_R(r) \ = f_Y\left(r^\alpha\right) \ \text{x} \ \ \alpha\, r^{\alpha-1} \tag{3.17}$$

$$f_R(r) = \frac{\mathrm{r}^{\alpha(n\ -\ 1)}}{(2\sigma^2)^n\ \Gamma(\mathrm{n})} \quad \exp\left[-\frac{r^\alpha}{2\sigma^2}\right] \alpha\, r^{\alpha-1} \tag{3.18}$$

$$f_R(r) = \frac{\alpha\ \mathrm{r}^{\alpha n\ -\ 1}}{(2\sigma^2)^n\ \Gamma(\mathrm{n})} \quad \exp\left[-\frac{r^\alpha}{2\sigma^2}\right] \tag{3.19}$$

Let us find E(r^k)

$$E\left(r^k\right) = \int_0^\infty r^k f_R\left(r\right) dr \tag{3.20}$$

$$E\left(r^k\right) = \int_0^\infty r^k \frac{\alpha r^{\alpha n-1}}{(2\sigma^2)^n \Gamma(n)} \exp\left(-\frac{r^\alpha}{2\sigma^2}\right) dr \tag{3.21}$$

$$E\left(r^k\right) = \frac{\alpha}{(2\sigma^2)^n \Gamma(n)} \int_0^\infty r^{\alpha n-1+k} \exp\left(-\frac{r^\alpha}{2\sigma^2}\right) dr \tag{3.22}$$

For solving integration of eq.(3.22), by using eq. (3.326.2) of Gradshteyn and Ryzhik (2007) given as $\int_0^\infty x^m e^{-\beta\, x^c}\ \mathrm{dx} \ = \ \frac{\Gamma(\gamma)}{\mathrm{c}\beta^\gamma}, \ \text{ where } \gamma = \frac{m+1}{c}$

On comparison we find, $x = r,\ m = \alpha n + k\text{ -1},\ c = \alpha,\ \beta = \frac{1}{2\sigma^2},\ \gamma = \frac{\alpha n + k\text{ -1}+1}{\alpha} = \frac{\alpha n + k}{\alpha} = n + \frac{k}{\alpha}$

Substituting these values in eq. (3.22), we get

$$E(r^k) = \frac{\alpha}{(2\sigma^2)^n\ \Gamma(\mu)} \frac{\Gamma\left(n+\frac{k}{\alpha}\right)}{\alpha\ \left(\frac{1}{2\sigma^2}\right)^{\left(n+\frac{k}{\alpha}\right)}} \tag{3.23}$$

$$E(r^k) = \frac{\Gamma\left(n+\frac{k}{\alpha}\right)}{\Gamma(n)}\left(2\sigma^2\right)^{\frac{k}{\alpha}} \tag{3.24}$$

Therefore $\quad E(r^\alpha) = 2\sigma^2 n \qquad (3.25)$

and $\quad E(r^{2\alpha}) = \frac{\Gamma(n+2)}{\Gamma(n)}(2\sigma^2)^2 = (2\sigma^2)^2(n+1)n \qquad (3.26)$

As we know Variance is defined as, $Var = E\left(r^2\right) - E^2\left(r\right) \qquad (3.27)$

Hence Normalized variance will be given as

$$Var = \frac{E\left(r^2\right) - E^2\left(r\right)}{E^2\left(r\right)} \tag{3.28}$$

Let μ be the inverse of the normalized variance of r^α, which will be given as

$$\mu = \frac{\mathrm{E}^2(r^\alpha)}{E(r^{2\alpha}) - E^2(r^\alpha)} = \frac{\left(2\sigma^2 n\right)^2}{\left(2\sigma^2\right)^2(n+1)n - \left(2\sigma^2 n\right)^2} = \frac{n^2}{n^2+n-n^2} = n \tag{3.29}$$

Also $\quad \hat{r} = \sqrt[\alpha]{E(r^\alpha)} = \sqrt[\alpha]{2\sigma^2 n} = \sqrt[\alpha]{2\sigma^2\mu} \qquad (3.30)$

Therefore, $\quad \hat{r}^\alpha = 2\sigma^2\mu \qquad (3.31)$

Substituting the values from eq. (3.29), (3.30), (3.31) in eq. (3.19)

We get, $\quad f_R(r) = \frac{\alpha\ r^{\alpha\mu-1}}{\left(\frac{\hat{r}}{\mu}\right)^\mu\ \Gamma(\mu)} \exp\left[-\frac{r^\alpha}{\hat{r}^\alpha/\mu}\right] \qquad (3.32)$

On simplifying we get, $\quad f_R(r) = \frac{\alpha\ \mu^\mu\ r^{\alpha\mu-1}}{\hat{r}^{\alpha\mu}\ \Gamma(\mu)} \exp\left[-\mu\frac{r^\alpha}{\hat{r}^\alpha}\right] \qquad (3.33)$

Eq. (3.33) is Probability Density Function (PDF) of α-μ fading channel.

where μ ≥ 0, is the inverse of the normalized variance of r^{α}, given as,

$$\mu = \frac{\mathrm{E}^2(R^{\alpha})}{V(R^{\alpha})} \tag{3.34}$$

$\Gamma(z) = \int_0^{\infty} t^{z-1}\, \mathrm{e}^{(-t)}\, \mathrm{dt}$ is the Gamma function.

E(.) and V(.) are the expectation and variance operators respectively.

Let $\rho = \frac{r}{\hat{r}}$, then α-μ PDF can be given as

$$f_{\rho}(\rho) = \frac{\alpha\ \mu^{\mu}\ \rho^{\alpha\mu-1}}{\Gamma(\mu)} \quad \exp\left[-\mu\rho^{\alpha}\right] \tag{3.35}$$

It can be observed from above equation that

(i) For $\alpha\mu < 1$, $f_{\rho}(\rho)$ tends to infinity as ρ approaches zero, and $f_{\rho}(\rho)$ decreases monotonically as ρ increases.

(ii) For $\alpha\mu = 1$, $f_{\rho}(\rho)$ has a non-zero and non-infinity value at origin and decreases monotonically to zero as ρ increases.

(iii) For αμ > 1, $f_{\rho}(\rho)$ is nil at origin and increases with increase of ρ to reach a maximum, then decreases as ρ increases.

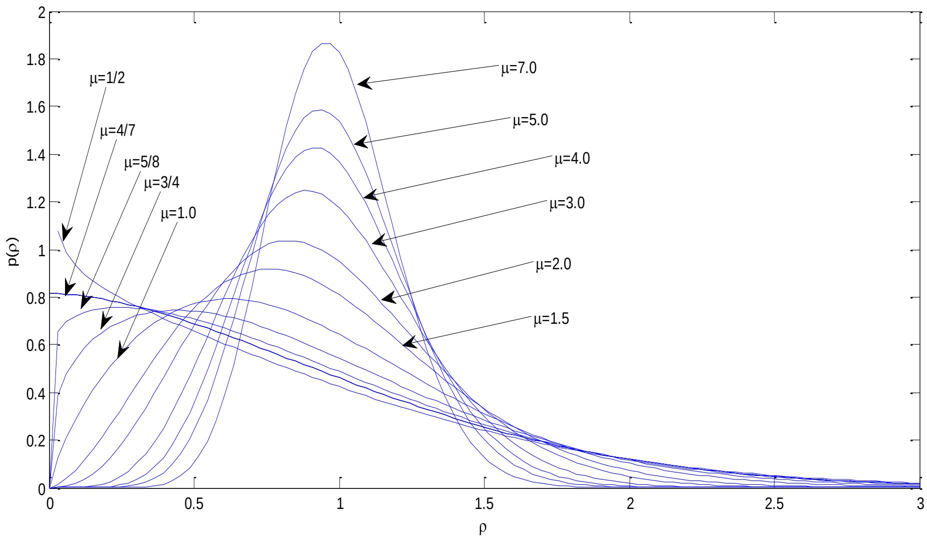

Fig. 3.2. Plot of PDF $f_{\rho}(\rho)$ versus ρ, keeping α constant (α=7/4) and varying μ

Fig. 3.2 shows plot of PDF $f_\rho(\rho)$ versus ρ, keeping α constant (α=7/4) for various values of μ. In Fig. 3.3 plot of PDF $f_\rho(\rho)$ versus ρ, keeping μ constant (μ=7/4) for various values of α is shown.

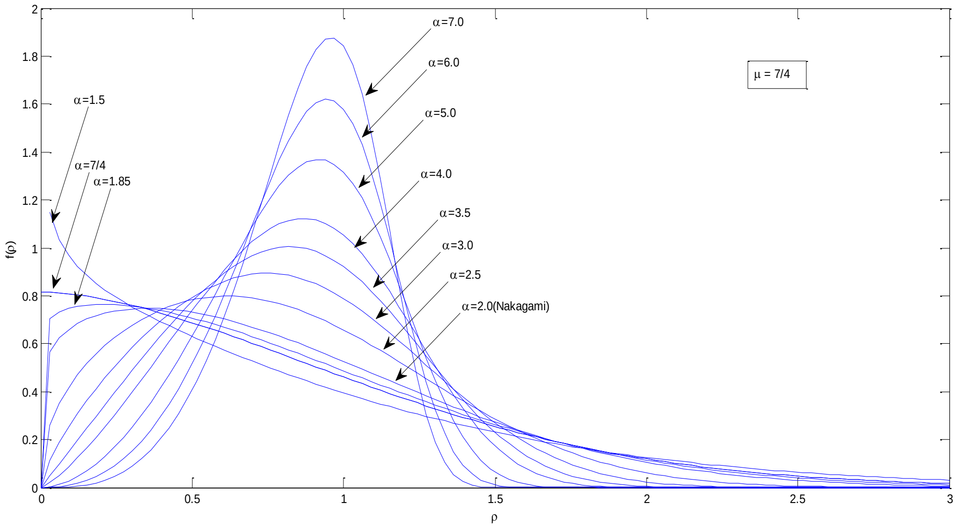

Fig.3.3. Plot of PDF $f_\rho(\rho)$ versus ρ, keeping μ constant (μ=7/4) and varying α.

3.3.2 Cumulative Distribution Function (CDF) of *α-u*:

CDF of α-μ distribution FR(r) can be given as, $F_R(r) = \int_0^\infty f_R(r)\, dr$ (3.36)

$$F_R(r) = \frac{\alpha\ \mu^\mu}{\hat{r}^{\alpha\mu}\ \Gamma(\mu)} \int_0^\infty r^{\alpha\mu\text{-}1}\ \exp\left[-\mu\frac{r^\alpha}{\hat{r}^\alpha}\right] dr \tag{3.37}$$

For solving integration of eq.(3.37), by using eq. (3.381.9) of Gradshteyn and Ryzhik (2007) given as

$$\int_u^\infty x^m e^{\text{-}\beta x^n}\ dx \ = \ \frac{\Gamma(v,\ \beta u^n)}{n\beta^v}, \qquad v = \frac{m+1}{n}$$

On comparison we find that

$$x = r,\ m = \alpha\mu - 1,\ \ \beta = \frac{\mu}{\hat{r}^\alpha},\ n = \alpha,\ v = \frac{m+1}{n} = \frac{\alpha\mu - 1 + 1}{\alpha} = \mu$$

Substituting these values in eq. (3.37), we get

$$F_R(r) = \frac{\alpha\ \mu^{\mu}}{\hat{r}^{\alpha\mu}\ \Gamma(\mu)} \quad \frac{\Gamma\left(\mu, \mu\left(\frac{r}{\hat{r}}\right)^{\alpha}\right)}{\alpha\ \frac{\mu^{\mu}}{\hat{r}^{\alpha\mu}}}, \tag{3.38}$$

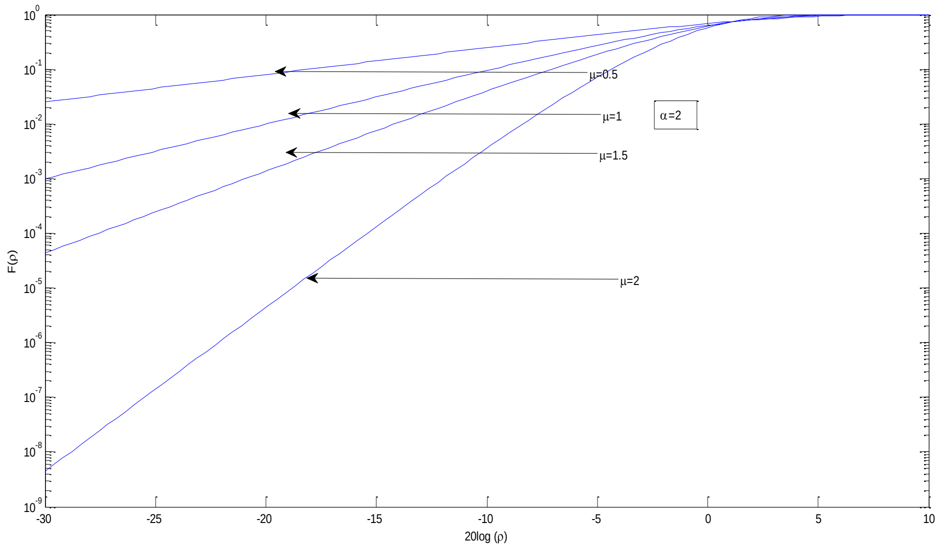

Fig. 3.4. Plot of CDF $F_\rho(\rho)$ versus 20 log(ρ), keeping α constant (α=2) and varying μ.

On simplifying we get, $$F_R(r) = \frac{\Gamma\left(\mu, \mu\left(\frac{r}{\hat{r}}\right)^{\alpha}\right)}{\Gamma(\mu)} \tag{3.39}$$

Since $\rho = \frac{r}{\hat{r}}$, CDF of $\alpha-\mu$ envelope will be $$F_P(\rho) = \frac{\Gamma\left(\mu, \mu\rho^{\alpha}\right)}{\Gamma(\mu)} \tag{3.40}$$

Fig.3.4. shows Plot of CDF $F_\rho(\rho)$ versus 20 log(ρ), keeping α constant (α=2) and varying μ.

3.3.3 The k^{th} moment of α-μ envelope:

The k^{th} moment of envelope R, $E(R^k)$ can be found as

$$E(R^k) = \int_0^{\infty} r^k \quad f_R(r) \quad dr \tag{3.41}$$

$$E(R^k) = \int_0^{\infty} r^k \quad \frac{\alpha\mu^{\mu} r^{\alpha\mu-1}}{\hat{r}^{\alpha\mu}\Gamma(\mu)} \exp\left(-\mu\frac{r^{\alpha}}{\hat{r}^{\alpha}}\right) \quad dr \qquad (3.42)$$

$$E(R^k) = \quad \frac{\alpha\mu^{\mu}}{\hat{r}^{\alpha\mu}\Gamma(\mu)} \int_0^{\infty} r^{\alpha\mu+k-1} \exp\left(-\mu\frac{r^{\alpha}}{\hat{r}^{\alpha}}\right) \quad dr \qquad (3.43)$$

For solving integration of eq.(3.43), by using eq. (3.326) of Gradshteyn and Ryzhik (2007) given as

$$\int_0^{\infty} x^m e^{-\beta x^n} \; dx \; = \; \frac{\Gamma(\gamma)}{n\beta^{\gamma}}, \quad where \ \gamma = \frac{m+1}{n}$$

On comparison we find that $x = r, \; m = \alpha\mu + k + 1, \; n = \alpha, \; \beta = \frac{\mu}{\hat{r}^{\alpha}}, \; \gamma = \frac{\alpha\mu + k - 1 + 1}{\alpha} = \frac{\alpha\mu + k}{\alpha} = \mu + \frac{k}{\alpha}$

Substituting these values in eq. (3.43), we get

$$E(R^k) = \quad \frac{\alpha \; \mu^{\mu}}{\hat{r}^{\alpha\mu} \; \Gamma(\mu)} \frac{\Gamma\left(\mu + \frac{k}{\alpha}\right)}{\alpha \; \left(\frac{\mu}{\hat{r}^{\alpha}}\right)^{\left(\mu+\frac{k}{\alpha}\right)}} \qquad (3.44)$$

Finally, k^{th} moment of envelope R, $E(R^k)$ is given as, $\quad E(R^k) = \quad \hat{r}^k \frac{\Gamma\left(\mu + \frac{k}{\alpha}\right)}{\mu^{\frac{k}{\alpha}} \; \Gamma(\mu)} \qquad (3.45)$

Let us define, $\quad w = \frac{r^{\alpha}}{\alpha}$ as a hyper-power

Then probability density function (PDF), $f_w(w)$, of w will be given as

$$f(w) = \frac{\mu^{\mu} \; w^{\mu-1}}{\overline{w}^{\mu} \; \Gamma(\mu)} \quad \exp\left[-\mu\frac{w}{\overline{w}}\right] \qquad (3.46)$$

The cumulative distribution function (CDF) $F_w(w)$ will be given as

$$F(w) = \int_0^{w} p(w) \;\; dw \qquad (3.47)$$

$$P(w) = \int_0^{w} \frac{\mu^{\mu} \; w^{\mu-1}}{\overline{w}^{\mu} \; \Gamma(\mu)} \exp\left[-\mu\frac{w}{\overline{w}}\right] dw \qquad (3.48)$$

$$P(w) = \frac{\mu^{\mu}}{\overline{w}^{\mu} \; \Gamma(\mu)} \int_0^{w} w^{\mu-1} \exp\left[-\mu\frac{w}{\overline{w}}\right] dw \qquad (3.49)$$

For solving integration of eq.(3.49), by using eq. (3.381) of Gradshteyn and Ryzhik (2007) given as

$$\int_0^u x^{\nu-1} e^{-\mu x} \, dx = \mu^{-\nu} \gamma(\nu, \mu u)$$

On comparison we find that $u = w, \ x = w, \ \nu = \mu, \ \mu = \frac{\mu}{\bar{w}}$

Therefore, substituting these values in eq. (3.49), we get

$$P(w) = \frac{\mu^\mu}{\bar{w}^\mu \, \Gamma(\mu)} \left(\frac{\mu}{\bar{w}}\right)^{-\mu} \gamma\left(\mu, \ \frac{\mu}{\bar{w}} w\right) \tag{3.50}$$

$$P(w) = \frac{1}{\Gamma(\mu)} \gamma\left(\mu, \ \mu \frac{w}{\bar{w}}\right) \tag{3.51}$$

$$P(r) = \frac{1}{\Gamma(\mu)} \Gamma\left(\mu, \ \mu\left(\frac{r}{\bar{r}}\right)^\alpha\right) \tag{3.52}$$

3.3.4 Signal to Noise Ratio (SNR) of α-μ

$$\text{Instantaneous SNR,} \quad \gamma = \bar{\gamma} \left(\frac{r}{\hat{r}}\right)^2 \tag{3.53}$$

$$\text{Where, } \bar{\gamma} = E\left[\hat{r}^2\right] \frac{E_b}{N_o} \tag{3.54}$$

and E_b/N_o is the energy per bit to the noise power spectral density ratio.

PDF of SNR, $f_\gamma(\gamma)$ can be found by carrying out random variable transformation as per Appendix –A6 on PDF of envelope given in eq. (3.33) reproduced as below:

$$f_R(r) = \frac{\alpha \ \mu^\mu \ \mathrm{r}^{\alpha\mu - 1}}{\hat{r}^{\alpha\mu} \ \Gamma(\mu)} \exp\left[-\mu \frac{r^\alpha}{\hat{r}^\alpha}\right]$$

We get,

$$f_\gamma(\gamma) = \frac{\alpha \ \mu^\mu \left[\hat{r}\left\{\frac{\gamma}{\bar{\gamma}}\right\}^{\frac{1}{2}}\right]^{\alpha\mu - 1}}{\hat{r}^{\alpha\mu} \ \Gamma(\mu)} \exp\left[-\mu \frac{\left\{\hat{r}\left(\frac{\gamma}{\bar{\gamma}}\right)^{1/2}\right\}^\alpha}{\hat{r}^\alpha}\right] x \ \frac{\hat{r}}{2\left(\gamma\bar{\gamma}\right)^{\frac{1}{2}}} \tag{3.55}$$

On solving, we get PDF of SNR

$$f_\gamma(\gamma) = \frac{\alpha\ \mu^\mu\ \gamma^{\frac{\alpha\mu}{2} - 1}}{2\ \Gamma(\mu)\ \bar{\gamma}^{\alpha\mu/2}} \quad e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} \tag{3.56}$$

3.3.5 Outage Probability (P_{out}) of α-μ

The outage probability is defined as the probability that the error rate exceeds a pre-defined value or equivalently, the received SNR drops below a pre-defined threshold (γ_{thr}).

P_{out} given by eq. (2.27) is $\quad P_{out} = \int_0^{\gamma_{thr}} f_\gamma(\gamma)\, d\gamma$

Substituting the value of $f_\gamma(\gamma)$ from eq. (3.56), we get

$$P_{out} = \int_0^{\gamma_{thr}} \frac{\alpha\ \mu^\mu\ \gamma^{\frac{\alpha\mu}{2} - 1}}{2\ \Gamma(\mu)\ \bar{\gamma}^{\alpha\mu/2}}\ \mathrm{e}^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}}\ d\gamma \tag{3.57}$$

$$P_{out} = \frac{\alpha\ \mu^\mu}{2\ \Gamma(\mu)\ \bar{\gamma}^{\alpha\mu/2}} \int_0^{\gamma_{\mathrm{thr}}} \gamma^{\frac{\alpha\mu}{2} - 1}\ \mathrm{e}^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}}\ d\gamma \tag{3.58}$$

For solving integration of eq.(3.58), by using eq. (3.381.8) of Gradshteyn and Ryzhik (2007) given as $\quad \int_0^u x^m e^{-\beta x^n}\ dx = \frac{\gamma(\nu,\ \beta u^n)}{n\beta^\nu} \qquad \nu = \frac{m+1}{n}$

On comparison we find that,

$$u = \gamma_{thr},\ x = \nu,\quad m = \frac{\alpha\mu}{2} - 1,\ \beta = \frac{\mu}{\bar{\gamma}^{\alpha/2}},\ n = \alpha/2,\ \nu = \frac{m+1}{n} = \frac{\frac{\alpha\mu}{2} - 1 + 1}{\alpha/2} = \mu$$

Therefore,

$$P_{out} = \frac{\alpha\ \mu^\mu}{2\ \Gamma(\mu)\ \bar{\gamma}^{\alpha\mu/2}} \quad \frac{\gamma\left(\dfrac{\frac{\alpha\mu}{2} - 1 + 1}{\alpha/2},\ \dfrac{\mu}{\bar{\gamma}^{\alpha/2}}\ \gamma_{thr}^{\alpha/2}\right)}{\dfrac{\alpha}{2}\left(\dfrac{\mu}{\bar{\gamma}^{\alpha/2}}\right)^\mu} \tag{3.59}$$

$$P_{out} = \frac{\alpha\ \mu^\mu}{2\ \Gamma(\mu)\ \bar{\gamma}^{\alpha\mu/2}} \quad \frac{\gamma\left(\mu,\ \mu\left(\dfrac{\gamma_{thr}}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\dfrac{\alpha}{2}\left(\dfrac{\mu}{\bar{\gamma}^{\alpha/2}}\right)^\mu} \tag{3.60}$$

$$P_{out} = \frac{\gamma_{thr}\left(\mu,\ \mu\left(\frac{\gamma_{thr}}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(u)} \tag{3.61}$$

Cumulative Distribution Function (CDF) of *α-μ* SNR:

The expression of P_{out} is also the expression for cumulative distribution function (CDF) of SNR for *α-μ* fading channel, thus

$$P(\gamma) = \frac{\gamma\left(\mu,\ \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(u)} \tag{3.62}$$

Where *γ(s, x)* is a Lower Incomplete Gamma function defined in Appendix –C2.

3.3.6 Moment Generating Function (MGF) of *α-μ*

MGF is very important characteristics of distribution function and helps in performance evaluation of wireless communication systems.

For example BER can be easily found if the MGF is known. For α-μ distribution, MGF can be found as

$$\mathrm{M}_{\gamma}(s) = E\left[e^{-s\gamma}\right] \tag{3.63}$$

$$\mathrm{M}_{\gamma}(s) = \int_{0}^{\infty} e^{-s\gamma} \frac{\alpha\ \mu^{\mu}\ \gamma^{\frac{\alpha\mu}{2}-1}}{2\ \Gamma(\mu)\ \bar{\gamma}^{\alpha\mu/2}}\ \mathrm{e}^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} d\gamma \tag{3.64}$$

$$\mathrm{M}_{\gamma}(s) = \frac{\alpha\ \mu^{\mu}}{2\ \Gamma(\mu)\ \bar{\gamma}^{\alpha\mu/2}} \int_{0}^{\infty} \gamma^{\frac{\alpha\mu}{2}-1} \mathrm{e}^{-s\gamma}\ \mathrm{e}^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} d\gamma \tag{3.65}$$

3.3.7 Bit Error Rate (BER) of *α-μ*:

The expression of BER for *α-μ* fading channel obtained through Moment Generating Function approach using Meijer's G-function as given by eq. (10) of Magableh and Matalgah (2009) is

$$P_e(\psi) = \frac{\alpha\mu^{\mu}}{\Gamma(\mu)\bar{\gamma}^{\alpha\mu/2}} \frac{\mathrm{k}^{\frac{1}{2}} l^{\frac{\alpha\mu-1}{2}}}{(2\pi)^{\frac{l+k}{2}}} I_{GI}(\psi) \tag{3.66}$$

$$\text{where, } I_{GI}(\psi) = \frac{l^{-\frac{1}{2}}\Gamma^2\left(\frac{1}{2}\right)}{2\psi^{\alpha\mu/2}} \times$$

$$G_{2l,k+l}^{k,2l}\left(\omega \left| \begin{array}{cc} I\left(l, \frac{1-\alpha\mu}{2}\right), & I\left(l, 1-\frac{\alpha\mu}{2}\right) \\ I(k,0), & I\left(l, -\frac{\alpha\mu}{2}\right) \end{array} \right.\right)$$

3.3.8 Level Crossing Rate (LCR) of α-μ Fading:

Level Crossing Rate (LCR) is defined as the rate at which the α-μ fading envelope, crosses a specified level in a positive-going or negative-going direction. The number of level crossings per second is given by

$$N_R(r) = \int_0^{\infty} \dot{r}\, f_{\dot{R},R}(\dot{r}, r) d\dot{r} \tag{3.67}$$

The upper dot denotes time derivative and

$f_{\dot{R},\mathrm{R}}(\dot{r}, r)$ is the joint density of $\dot{\mathrm{R}}$ and R.

Let envelope of *α-μ* be *R* and envelope of Nakagami-*m* be R_N. Then

$$R^{\alpha} = R_N^2 \tag{3.68}$$

Differentiating w.r.t. time we get

$$\alpha R^{\alpha-1}\dot{R} = 2R_N\dot{R}_N \tag{3.69}$$

$$\dot{R} = \frac{2}{\alpha} R^{1-\frac{\alpha}{2}} \dot{R}_N \tag{3.70}$$

Since $\dot{R}_N$ is known to be zero-mean Gaussian distribtuted with variance $\dot{\sigma}_N^2$.

Therefore, the probability density function $f_{\dot{R}/R}(\dot{r}/r)$ of $\dot{R}$ given R is also zero-mean Gaussian distribtuted with variance $\dot{\sigma}^2$.

Since, from above we have

$$\dot{\sigma}^2 = \left(\frac{2}{\alpha} r^{1-\frac{\alpha}{2}}\right)^2 \dot{\sigma}_N^2 \tag{3.71}$$

$$\dot{\sigma}^2 = \frac{4}{\alpha^2} r^{2-\alpha} \dot{\sigma}_N^2 \tag{3.72}$$

The joint density function $f_{\dot{R},\,R}(\dot{r}, r) = f_{\dot{R}/R}(\dot{r}/r)\, f_R(r)$

$$f_{\dot{R},R}(\dot{r}, r) = \frac{1}{\sqrt{2\pi}\dot{\sigma}} \exp\left(-\frac{\dot{r}^2}{2\dot{\sigma}^2}\right) \text{ x } \frac{\alpha\ \mu^{\mu}\ r^{\alpha\mu - 1}}{\hat{r}^{\alpha\mu}\ \Gamma(\mu)} \quad \exp\left[-\mu\frac{r^{\alpha}}{\hat{r}^{\alpha}}\right] \tag{3.73}$$

$$N_R(r) = \int_0^{\infty} \dot{r}\, f_{\dot{R},R}(\dot{r}, r) d\dot{r}$$

$$= \int_0^{\infty} \dot{r} \frac{1}{\sqrt{2\pi}\dot{\sigma}} \exp\left(-\frac{\dot{r}^2}{2\dot{\sigma}^2}\right) x \frac{\alpha\ \mu^{\mu}\ r^{\alpha\mu - 1}}{\hat{r}^{\alpha\mu}\ \Gamma(\mu)} \quad \exp\left[-\mu\frac{r^{\alpha}}{\hat{r}^{\alpha}}\right] d\dot{r} \tag{3.74}$$

$$N_R(r) = \frac{1}{\sqrt{2\pi}\dot{\sigma}}\ x \frac{\alpha\ \mu^{\mu}\ r^{\alpha\mu - 1}}{\hat{r}^{\alpha\mu}\ \Gamma(\mu)} \quad \exp\left[-\mu\frac{r^{\alpha}}{\hat{r}^{\alpha}}\right] \int_0^{\infty} \dot{r}\ \exp\left(-\frac{\dot{r}^2}{2\dot{\sigma}^2}\right) d\dot{r} \tag{3.75}$$

$$N_R(r) = \frac{1}{\sqrt{2\pi}\dot{\sigma}}\ x\, f_R(r) \int_0^{\infty} \dot{r}\ \exp\left(-\frac{\dot{r}^2}{2\dot{\sigma}^2}\right) d\dot{r} \tag{3.76}$$

For solving integration of eq.(3.76), by using eq. (3.326) of Gradshteyn and Ryzhik (2007) given as $\int_0^{\infty} x^m e^{-\beta x^n}\ dx = \frac{\Gamma(\gamma)}{n\beta^{\gamma}}$, where $\gamma = \frac{m+1}{n}$

On comparison we find that, $x = \dot{r},\ m = 1,\ n = 2,\ \beta = \frac{1}{2\dot{\sigma}^2},\ \gamma = \frac{m+1}{n} = \frac{2}{2} = 1$

Therefore, $N_R(r) = \frac{1}{\sqrt{2\pi}\dot{\sigma}}\ x\, f_R(r)\ \frac{\Gamma(1)}{2\left(\frac{1}{2\dot{\sigma}^2}\right)}$ (3.77)

$$N_R(r) = \frac{\dot{\sigma}\, f_R(r)}{\sqrt{2\pi}} \tag{3.78}$$

Further, variance for Nakagami-*m* is given as

$$\dot{\sigma}_R^2 = \left(\frac{\omega}{2}\right)^2 \frac{\left(E\left(R_N^2\right)\right)}{m} \tag{3.79}$$

Where ω is the maximum Doppler shift in radians per second.

Since, $R_N^2 = R^\alpha$ therefore, $E\left(R_N^2\right) = E\left(R^\alpha\right)$

Also, $\frac{E^2\left(R_N^2\right)}{V\left(R_N^2\right)} = \frac{E^2\left(R^\alpha\right)}{V\left(R^\alpha\right)}$

Therefore, $E\left(R_N^2\right) = \hat{r}^\alpha$ and $m = \mu$

Hence, $\dot{\sigma}_N^2 = \left(\frac{\omega}{2}\right)^2 \left(\frac{\hat{r}^\alpha}{\mu}\right)$ (3.80)

$$\dot{\sigma}^2 = \left(\frac{4}{\alpha^2}\right) r^{2-\alpha} \left(\frac{\omega}{2}\right)^2 \left(\frac{\hat{r}^\alpha}{\mu}\right) \tag{3.81}$$

$$\dot{\sigma}^2 = \left(\frac{\omega}{\alpha}\right)^2 \left(\frac{\hat{r}^\alpha r^{2-\alpha}}{\mu}\right) \tag{3.82}$$

Since, $f_{\dot{R},R}(\dot{r},r) = \frac{1}{\sqrt{2\pi}\dot{\sigma}} \exp\left(-\frac{\dot{r}^2}{2\dot{\sigma}^2}\right) \times \frac{\alpha\ \mu^\mu\ r^{\alpha\mu - 1}}{\hat{r}^{\alpha\mu}\ \Gamma(\mu)}\ \exp\left[-\mu \frac{r^\alpha}{\hat{r}^\alpha}\right]$ (3.83)

Therefore,

$$f_{\dot{R},R}(\dot{r},r) = \frac{1}{\sqrt{2\pi}\,\frac{\omega}{\alpha}\left(\frac{\hat{r}^\alpha r^{2-\alpha}}{\mu}\right)^{\frac{1}{2}}} \exp\left(-\frac{\dot{r}^2}{2\left(\frac{\omega}{\alpha}\right)^2\left(\frac{\hat{r}^\alpha r^{2-\alpha}}{\mu}\right)}\right) \times \frac{\alpha\ \mu^\mu\ r^{\alpha\mu - 1}}{\hat{r}^{\alpha\mu}\ \Gamma(\mu)} \exp\left[-\mu \frac{r^\alpha}{\hat{r}^\alpha}\right] \tag{3.84}$$

$$f_{\dot{R},R}(\dot{r},r)=\frac{\alpha^2\mu^{\mu+0.5}r^{\alpha\mu-1-1+\frac{\alpha}{2}}}{\sqrt{2\pi}\;\omega\;\hat{r}^{\alpha\mu+\frac{\alpha}{2}}\;\Gamma(\mu)}\exp\left(-\frac{\dot{r}^2}{2\left(\frac{\omega}{\alpha}\right)^2\left(\frac{\hat{r}^{\alpha}r^{2-\alpha}}{\mu}\right)}\right)\times\exp\left[-\mu\frac{r^{\alpha}}{\hat{r}^{\alpha}}\right] \tag{3.85}$$

We have LCR from eq. (3.67) as

$$N_R(r)=\int_0^{\infty} r\; f_{\dot{R},R}(\dot{r},r)\;d\dot{r}$$

On substituting the values from eq. (3.85), in eq. (3.67) we get,

$$N_R(r)=\frac{\alpha^2\mu^{\mu+0.5}r^{\alpha(\mu+0.5)-2}}{\sqrt{2\pi}\;\omega\;\hat{r}^{\alpha(\mu+0.5)}\;\Gamma(\mu)}\exp\left(-\mu\frac{r^{\alpha}}{\hat{r}^{\alpha}}\right)\int_0^{\infty}\dot{r}\exp\left(-\frac{\mu\;\alpha^2\;r^{\alpha-2}\;\dot{r}^2}{2\omega^2\;\hat{r}^{\alpha}}\right)d\dot{r} \tag{3.86}$$

For solving integration of eq.(3.86), by using eq. (3.326.2) of Gradshteyn and Ryzhik (2007) given as $\int_0^{\infty} x^m e^{-\beta\,x^n}\;dx\;=\;\frac{\Gamma(\gamma)}{n\beta^{\gamma}},\quad \text{where } \gamma=\frac{m+1}{n}$

On comparison we find that

$$x=\dot{r},\; m\;=\;1,\;\; \beta=\frac{\mu\alpha^2r^{\alpha-2}}{2\omega^2\hat{r}^{\alpha}},\; n=2,\; \gamma=\frac{m+1}{n}=\frac{2}{2}=1$$

Therefore,

$$N_R(r)=\frac{\alpha^2\mu^{\mu+0.5}r^{\alpha(\mu+0.5)-2}}{\sqrt{2\pi}\;\omega\;\hat{r}^{\alpha(\mu+0.5)}\;\Gamma(\mu)}\exp\left(-\mu\frac{r^{\alpha}}{\hat{r}^{\alpha}}\right)\;\frac{1}{2\left(\frac{\mu\alpha^2r^{\alpha-2}}{2\omega^2\hat{r}^{\alpha}}\right)^1} \tag{3.87}$$

Finally LCR is given as

$$N_R(r)=\frac{\omega\;\mu^{\mu-0.5}\rho^{\alpha(\mu+0.5)}}{\sqrt{2\pi}\;\;\Gamma(\mu)\;\exp\;(\mu\rho^{\alpha})},\quad \text{where } \rho=\frac{r}{\hat{r}} \tag{3.88}$$

3.3.9 Average Fade Duration (AFD) of α-μ Fading:

AFD is the average period of time for which the received signal is below a specified level R. AFD is given as

$$\text{AFD is given as} \qquad T_R(r) = \frac{F_R(r)}{N_R(r)} \tag{3.89}$$

$$T_R(r) = \frac{\sqrt{2\pi}\;\; F_R(r)}{\dot{\sigma}\;\; f_R(r)} \tag{3.90}$$

$$T_R(r) = \frac{F_R(r)}{N_R(r)} = \frac{\dfrac{\Gamma\left(\mu, \mu\rho^{\alpha}\right)}{\Gamma\left(\mu\right)}}{\dfrac{\omega\mu^{\mu-0.5}\rho^{\alpha(\mu-0.5)}}{\sqrt{2\pi}\Gamma(\mu)\exp(\mu\rho^{\alpha})}} \tag{3.91}$$

Finally AFD is given as

$$T_R(r) = \frac{\sqrt{2\pi}\;\; \Gamma\left(\mu, \mu\rho^{\alpha}\right)\;\; \exp\left(\mu\rho^{\alpha}\right)}{\omega\; \mu^{\mu-0.5}\rho^{\alpha(\mu-0.5)}} \tag{3.92}$$

3.3.10. Amount of Fading (AF) for α-μ distribution:

For evaluating the AF defined by eq. (2.30), first we will evaluate n^{th} moment of SNR (γ) of α-μ fading. Since, the PDF of instantaneous SNR (γ) of α-μ is given by eq.(3.56) as

$$f_{\gamma}(\gamma) = \frac{\alpha\;\mu^{\mu}\;\gamma^{\frac{\alpha\mu}{2}-1}}{2\;\Gamma(\mu)\;\bar{\gamma}^{\alpha\mu/2}} \quad e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}}, \;\text{ where }\; \bar{\gamma} = \frac{\mu^{\frac{2}{\alpha}}\;\Gamma\left(\mu\right)}{\Gamma\left(\mu + \dfrac{2}{\alpha}\right)}\frac{E_b}{N_0}, \tag{3.93}$$

$\frac{E_b}{N_0}$ is the energy per bit to the noise power spectral density ratio.

Hence, n^{th} moment of SNR (γ) of α-μ fading will be

$$E\left(\gamma^n\right) = \int_0^{\infty} \gamma^n f_{\gamma}\left(\gamma\right) d\gamma \tag{3.94}$$

$$E\left(\gamma^n\right)=\int_0^\infty \gamma^n \frac{\alpha\ \mu^\mu\ \gamma^{\frac{\alpha\mu}{2}-1}}{2\ \Gamma(\mu)\ \bar{\gamma}^{\alpha\mu/2}}\quad e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}}\ d\gamma \tag{3.95}$$

$$E\left(\gamma^n\right)=\frac{\alpha\ \mu^\mu}{2\ \Gamma(\mu)\ \bar{\gamma}^{\alpha\mu/2}}\quad \int_0^\infty \gamma^{\frac{\alpha\mu}{2}-1}\ \mathrm{e}^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}}\ d\gamma \tag{3.96}$$

For solving integration of eq.(3.96), by using eq. (3.381.9) of Gradshteyn and Ryzhik (2007) given as $\int_u^\infty x^m e^{-\beta\, x^k}\ dx\ =\ \frac{\Gamma(v,\beta u^k)}{k\beta^v},\quad \text{where } v=\frac{m+1}{k}$

On comparison we find that

$$u=0, x=\gamma,\ m\ =\ \frac{\alpha\mu}{2}+n-1,\ k=\ \frac{\alpha}{2},\ \beta=\ \frac{\mu}{\bar{\gamma}^{\alpha/2}}\ ,\ v=\frac{\frac{\alpha\mu}{2}+n-1+1}{\frac{\alpha}{2}}=\frac{\frac{\alpha\mu}{2}+n}{\frac{\alpha}{2}}$$

Therefore, from eq.(3.96) we get,

$$E\left(\gamma^n\right)=\frac{\alpha\ \mu^\mu}{2\ \Gamma(\mu)\ \bar{\gamma}^{\alpha\mu/2}}\quad \frac{\Gamma\left(\frac{\frac{\alpha\mu}{2}+n}{\frac{\alpha}{2}},0\right)}{\frac{\alpha}{2}\left(\frac{\mu}{\bar{\gamma}^{\alpha/2}}\right)^{\frac{\frac{\alpha\mu}{2}+n}{\frac{\alpha}{2}}}} \tag{3.97}$$

On solving further eq.(3.97), we get

$$E\left(\gamma^n\right)=\frac{\mu^{-\frac{2n}{\alpha}}\ \bar{\gamma}^n\ \Gamma\left(\frac{\frac{\alpha\mu}{2}+n}{\alpha/2}\right)}{\Gamma(\mu)} \tag{3.98}$$

Now, with help of eq. (2.30) the amount of fading (AF) for α-μ fading will be

$$A\,F=\ \frac{\text{var}\ \gamma_t}{(E[\gamma_t])^2}\ =\frac{E(\gamma_t^2)-(E[\gamma_t])^2}{(E[\gamma_t])^2}$$

$$
A\,F = \frac{\dfrac{\mu^{-\frac{2\times 2}{\alpha}}\ \bar{\gamma}^{2}\ \Gamma\left(\dfrac{\frac{\alpha\mu}{2}+2}{\alpha/2}\right)}{\Gamma(\mu)} \ - \ \left[\dfrac{\mu^{-\frac{2\times 1}{\alpha}}\ \bar{\gamma}\ \Gamma\left(\dfrac{\frac{\alpha\mu}{2}+1}{\alpha/2}\right)}{\Gamma(\mu)}\right]^{2}}{\left[\dfrac{\mu^{-\frac{2\times 1}{\alpha}}\ \bar{\gamma}\ \Gamma\left(\dfrac{\frac{\alpha\mu}{2}+1}{\alpha/2}\right)}{\Gamma(\mu)}\right]^{2}} \tag{3.99}
$$

$$
A\,F = \frac{\Gamma(\mu)\ \Gamma\left(\dfrac{\alpha\mu+4}{\alpha}\right) \ - \ \Gamma^{2}\left(\dfrac{\alpha\mu+2}{\alpha}\right)}{\Gamma^{2}\left(\dfrac{\alpha\mu+2}{\alpha}\right)} \tag{3.100}
$$

$$
A\,F = \frac{\Gamma(\mu)\ \Gamma\left(\dfrac{\alpha\mu+4}{\alpha}\right)}{\Gamma^{2}\left(\dfrac{\alpha\mu+2}{\alpha}\right)} \ - \ 1 \tag{3.101}
$$

CHAPTER 4

PERFORMANCE OF α-μ CHANNEL

4.1 Performance Analysis of *α-μ* Fading Channel:

The PDF of fading envelope of α-μ channel as defined by eq. (3.33) is simulated using MATLAB environment. First, we developed the algorithm used for simulating the PDF of α-μ fading envelope as given in Appendix –B, to ensure that the simulated results match with the analytical results. The result presented in fig.4.1 and fig.4.2 are simulated and analytical results obtained by varying the values of α and μ as given below:

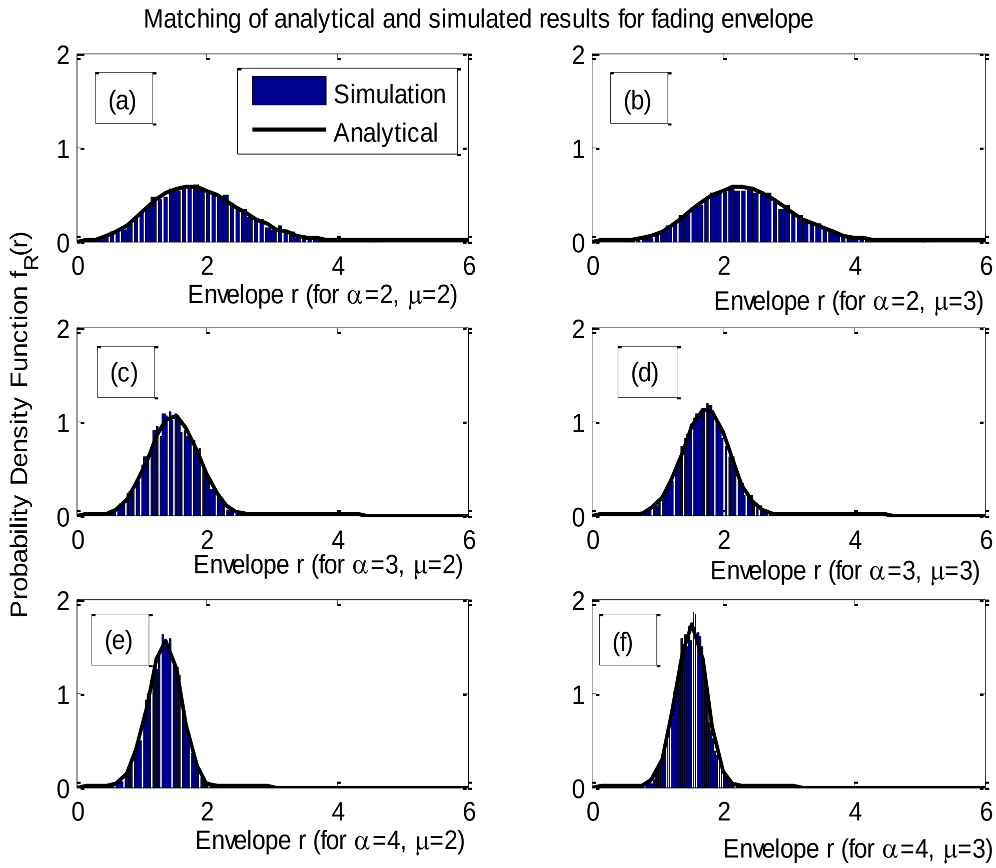

Fig. 4.1. Matching of analytical & simulated results of fading amplitude PDF for different α and μ (a) α=2 and μ=2, (b) α=2 and μ=3, (c) α=3 and μ=2, (d) α=3 and μ=3, (e) α=4 and μ=2, (f) α=4 and μ=3

Case –I: Fig.4.1.(a), (c), (e); varying α from 2 to 4, and μ =2.

Case –II: Fig.4.1.(b), (d), (f); varying α from 2 to 4, and μ =3.

Case –III: Fig.4.2.(a), (c), (e); varying α from 2 to 4, and μ =4.

Case –IV: Fig.4.2.(b), (d), (f); varying α and μ for special cases of Rayleigh, Exponential and Nakagami-*m*.

It is seen that the simulated result matches with analytical curve. It is observed that by increase in α, the base of the PDF curve shrinks which means variance decreases and vice versa. Whereas by increase in μ the curve becomes more peaky which means probability of mean increases.

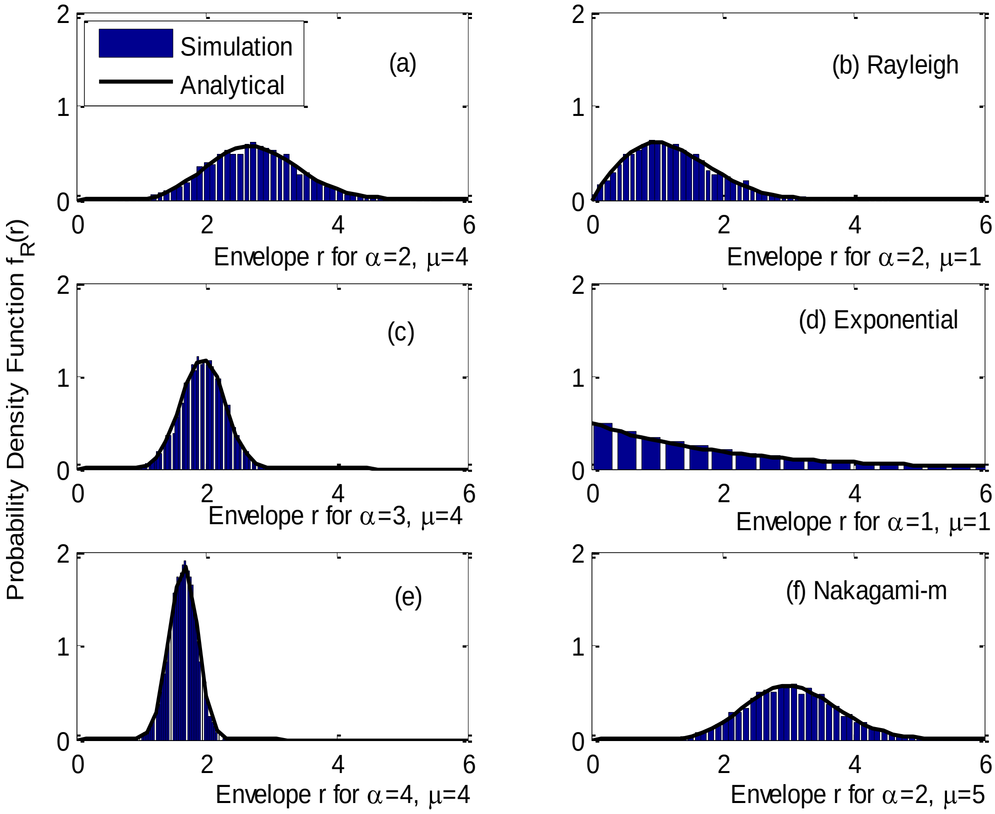

Fig. 4.2. Matching of analytical & simulated results of fading amplitude PDF for different α and μ (a) α=2 and μ=4, (b) α=2 and μ=1, (c) α=3 and μ=4, (d) α=1 and μ=1, (e) α=4 and μ=4, (f) α=2 and μ=5

Outage performance and BER for BPSK modulated α-μ fading channel is shown in fig. 4.3 and fig.4.4 respectively. In these simulations 100000 samples have been considered for each combination of α and μ. It is observed that for a given value of μ, at zero dB SNR outage is same irrespective of α but outage decreases with increase in α at higher value of SNR. BER for α-μ fading channel is shown in fig.4.4, following cases of BER curves are illustrated:

Case –I: Varying α from 2 to 4, and keeping μ =2. It is seen that at 7 dB SNR the curves crosses each other.

Case –II: Varying α from 2 to 4, and keeping μ =3. It is seen that at 8 dB SNR the curves crosses each other.

Case –III: Varying α from 2 to 4, and keeping μ =4. It is seen that at 9 dB SNR the curves crosses each other.

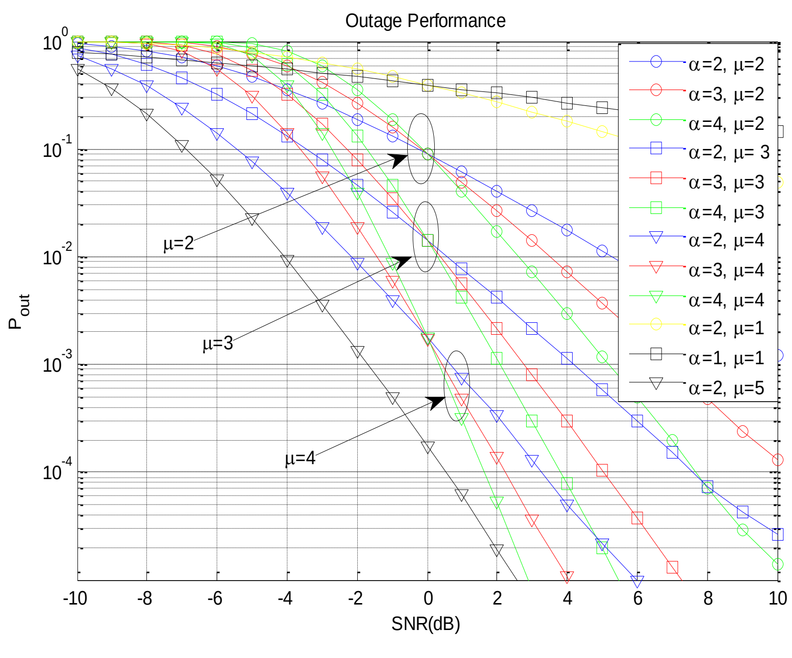

Fig. 4.3. Outage performance of α-μ fading channel

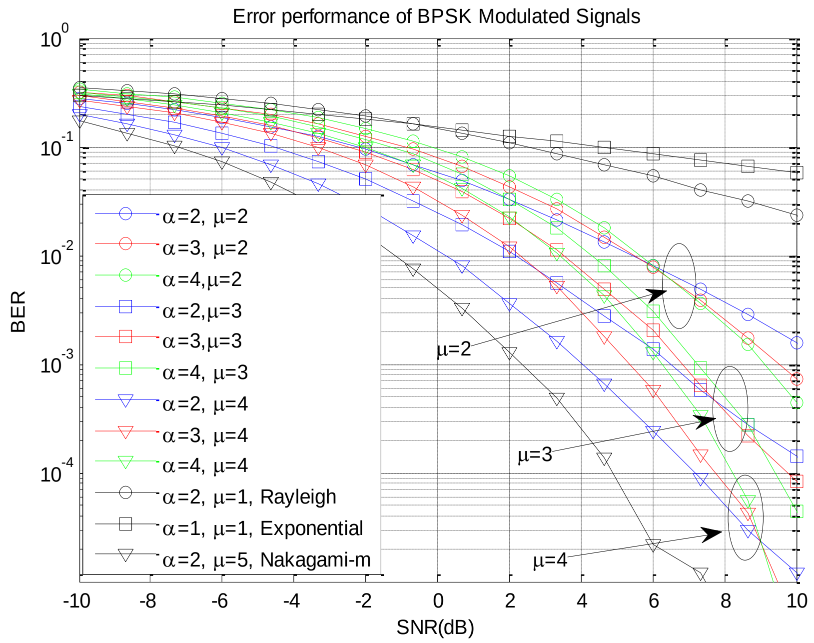

Fig. 4.4. Bit Error Rate of α-μ fading channel

Let us call the SNR on these crossing points as critical SNR (i.e. SNR_c) for a given value of μ. It is observed that if SNR of the channel is below SNR_c then BER is proportional to α but for SNR greater than SNR_c the BER is inversely proportional to α. In fig.4.5 and fig.4.6 BER and outage curves are illustrated for α varying from 1 to 7, while keeping μ =1. Inference can be drawn from fig.4.5 and fig.4.6 that after certain level, further increase in α there is no significant improvement in BER and outage respectively. In fig.4.7 and fig. 4.8 μ hass been varied from 1 to 7 and α is equal to 1, BER and outage plot are shown. It is observed that as the value of μ increases there is improvement in BER and outage. At very high level of μ, further improvement in BER and outage does not happen.

In this analysis PDF, Outage and BER of α-μ fading channel have been illustrated. The effect of α and μ parameters variation on BER and outage is discussed. The result obtained in this letter has been published in a research paper titled ***"Performance Analysis of Wireless Link operating in α-μ Fading Channel***" in International Journal of Emerging

Technology and Advanced Engineering (IJETAE), ISSN 2250-2459, vol. 5, iss. 6, Jun 2015, pp.540-547.

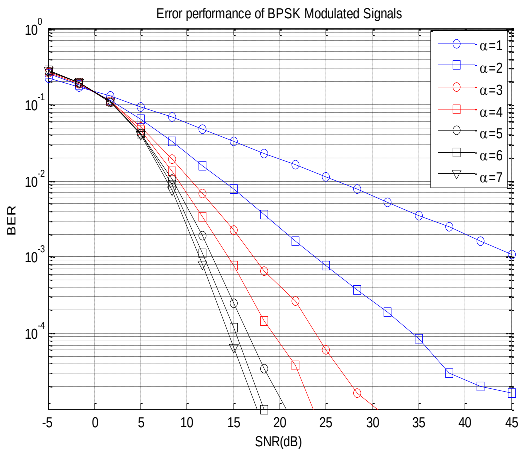

Fig. 4.5. BER of α-μ fading channel α varied from 1 to 7, keeping μ=1

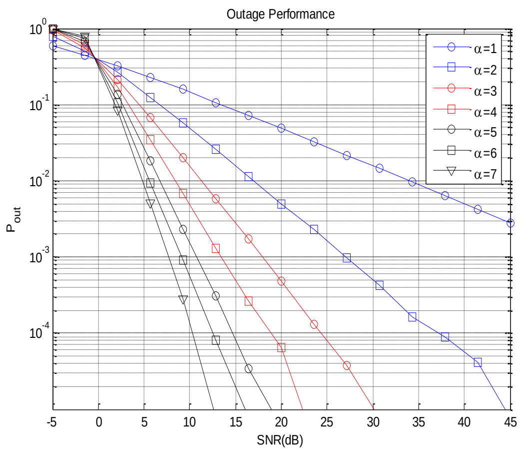

Fig. 4.6. Outage of α-μ fading channel α varied from 1 to 7, keeping μ=1

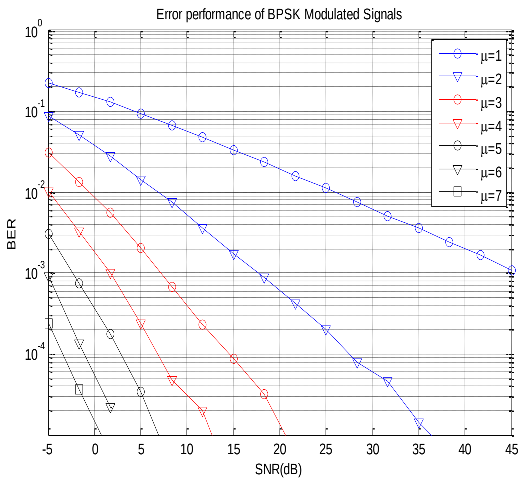

Fig. 4.7. BER of α-μ fading channel μ varied from 1 to 7, keeping α=1

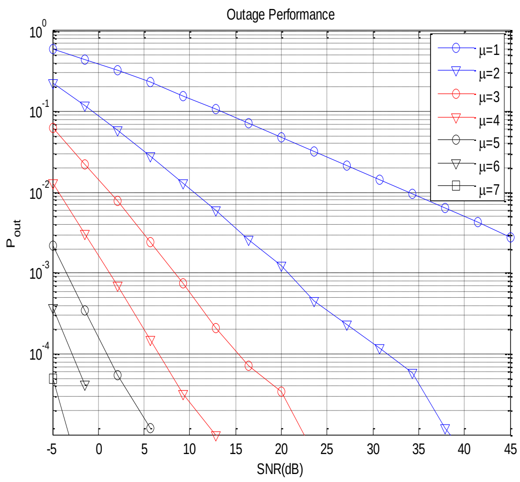

Fig. 4.8. Outage of α-μ fading channel μ varied from 1 to 7, keeping α=1

4.2 Performance of Coded α-μ Channel:

Shannon (1948) demonstrated for AWGN channel, that error induced by a noisy channel can be reduced to any desired level without sacrificing the rate of information transfer, by proper encoding of the information. Shannon channel capacity formula is given as

$$C = B\log_2\left(1+\frac{P}{N_0 B}\right) = B\log_2\left(1+\frac{S}{N}\right) \tag{4.1}$$

Where C is the channel capacity (bits per second), B is the transmission bandwidth (Hz), P is the received signal power (W), and N_0 is the single-sided noise power density (W/Hz). Also, $P = E_b.R_b$, where E_b is the average bit energy and R_b is the transmission bit rate.

The increasing practical usage of digital communication systems has led of late to a great deal of work devoted to coding for fading channels. Biglieri Ezio, et. al. (2000) have described a detailed survey of coding for fading channels. For fading channels where nonlinearities, Doppler shifts, fading, shadowing, and interference from other users make the channel completely different than Gaussian, hence the concepts developed for the Gaussian channel may not be valid anymore, and a fresh look at the channel coding design philosophies is needed.

4.2.1 Error Correcting Codes:

When a digital signal is transmitted, the signal gets distorted because of noise; this means that a 0 may change to 1 or a 1 may change to 0. Therefore, it is essential to detect and correct the error. Coding helps bit errors introduced by transmission of a modulated signal through a wireless channel to be either detected or corrected by a decoder in the receiver. The error correction enhances the performance on the cost of decrease in data rate or increase in signal bandwidth. The general idea for achieving error detection and correction is to add some redundancy i.e. some extra data known as parity bits to a message, which receivers can use to check correctness of the transmitted data and to recover the same. The data bits along with the parity bits form a code word. The parity data are derived from the data bits by some deterministic algorithm. For error detection, a receiver can simply apply the same algorithm to the received data bits and compare its output with the received check bits, if the values do not match, an error has occurred at some point during the transmission.

The transmission errors are basically two types namely the random errors and burst errors. The errors that occur in purely random manners are known as random errors. They are independent in nature. When more than one data bits change their state simultaneously, due to impulse noise, the error introduced is called as burst errors.

Error control for data integrity can be achieved by using the techniques called Automatic Repeat Request (ARQ) or Forward Error Correction (FEC). In ARQ receiver requests for retransmission of complete or part of message if it finds error, hence requires a feedback channel for sending the request. Whereas in FEC, the feedback channel is not required. There are two types of error correcting codes, Block codes and Convolution codes. Convolution codes require memory, whereas Block codes are memory less.

4.2.2 Block Codes:

In block codes one of the $M=2^k$ messages, each having binary sequence of length k, called message bits or information sequence, is mapped to a binary sequence of length n, called the codeword, where $n > k$. Generation of n-bit block code is shown in fig. 4.9.

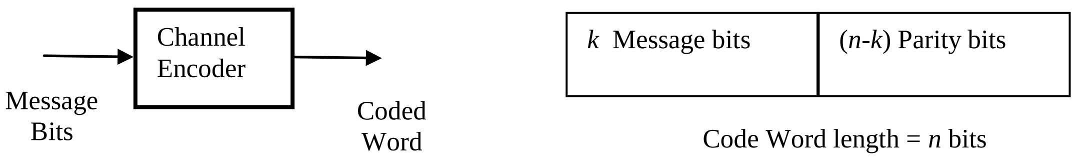

Fig. 4.9. Generation of n-bit block code

In an (n, k) binary block code, the rate of code is given as $R_c = k/n$ information bits per codeword symbol. If the codeword symbols are transmitted across the channel at a rate of R_s symbols per second, then information rate associated with an (n, k) block code is $R_b = R_c R_s = (k/n)R_s$ bits per second. Thus the block coding reduces the data rate.

4.2.3 Generator Matrix in a Linear Block Code:

The generator matrix is a compact description of how codewords are generated from information bits in a linear block code. Goldsmith (2005) explained for designing a generator matrix, by taking a (n, k) code with k information bits, denoted as

$$U_i = [u_{i1}, ..., u_{ik}] \tag{4.2}$$

These k information bits are encoded into the codeword

$$C_i = [c_{i1}, ..., c_{in}] \tag{4.3}$$

Let us represent the coding operation as a set of n equations given as

$$c_{ij} = u_{i1}g_{1j} + u_{i2}g_{2j} + ... + u_{ik}g_{1k}; \quad j = 1, ..., n. \tag{4.4}$$

Here g_{ij} is binary (0 or 1) and binary multiplication is used. Writing these n equations in matrix form.

$$C_i = U_i \; G \tag{4.5}$$

the $k \times n$ generator matrix G for the code is given as

$$G = \begin{pmatrix} g_{11} & \cdots & g_{1n} \\ \vdots & \ddots & \vdots \\ g_{k1} & \cdots & g_{kn} \end{pmatrix} \tag{4.6}$$

If we denote the l^{th} row of $\boldsymbol{G}$ as

$$g_l = [g_{l1}, ..., g_{ln}] \tag{4.7}$$

Then any codeword $\boldsymbol{C_i}$ will be

$$C_i = u_{i1}g_1 + u_{i2}g_2 + ... + u_{ik}g_k \tag{4.8}$$

A linear block is defined with help of a generator matrix of the form

$$G=\left[\mathrm{I}_k \,\middle|\, \mathrm{P}\right]=\left[\begin{array}{cccc|cccc} 1 & 0 & \cdots & 0 & p_{11} & p_{12} & \cdots & p_{1(n-k)} \\ 0 & 1 & \cdots & 0 & p_{21} & p_{22} & \cdots & p_{2(n-k)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & p_{k1} & p_{k2} & \cdots & p_{k(n-k)} \end{array}\right] \tag{4.9}$$

Where $\mathbf{I}_k$ is the $k \times k$ identity matrix and **P** is a $k \times (n\text{-}k)$ matrix which determines the redundant bits to be used for error detection and correction.

The codeword output from encoder is given as

$$\mathrm{C}_i = \mathrm{U}_i G = U_i \left[\mathrm{I}_k \,|P\right] = \left[u_{i1}, \ldots\ldots, u_{ik}, p_1, \ldots.., p_{(n-k)}\right] \tag{4.10}$$

Here in the codeword first k bits are the information bits and the last $(n\text{-}k)$ bits are the parity bits obtained from the information bits as

$$p_j = u_{i1} p_{1j} + u_{i2} p_{2j} + \ldots\ldots + u_{ik} p_{kj}; \quad j = 1, \ldots.., (n-k) \tag{4.11}$$

In linear block code, the probability of error depends on the decoder using ‘hard decisions’ or ‘soft decisions’. In hard decision decoding (HDD), each encoded bit is demodulated as 0 or 1. For BPSK case, the received symbol is decoded as 1 if it is closure to $+\sqrt{E}$ or as 0 if it is closure to $-\sqrt{E}$. This form of demodulation removes the information which can be used by channel decoder. When these distances are used in the channel decoder it is called soft decision decoding (SDD).

4.2.4 Cyclic Code:

Cyclic codes are a subclass of linear block codes. Additional structure built in cyclic code makes their decoding simpler than block codes. In cyclic code all codewords in a given code are cyclic shifts of one another.

If in a given code a codeword is

$$\mathrm{C} = \left[c_0, c_1, ..., c_{n-1} \right] \tag{4.12}$$

Then a cyclic shift by 1, is also a codeword. i.e.

$$\mathrm{C}^{(1)} = \left[c_{n-1}, c_0, c_1, ..., c_{n-2} \right] \tag{4.13}$$

In general, any cyclic shift is also a codeword. i.e.

$$\mathrm{C}^{(\mathrm{i})} = \left[c_{n-i}, c_{n-i+1}, ..., c_{n-i-1} \right] \tag{4.14}$$

The cyclic nature of cyclic codes allows their encoding and decoding functions to be of lower complexity than the matrix multiplications carried out with encoding and decoding for general linear block codes. The cyclic codes is extensively used in practice. The Bose-Chadhuri-Hocqueghem (BCH) code and Reed-Solomon (RS) codes belongs to the class of Cyclic code. Cyclic codes are generated with help of a generator polynomial in place of a generator matrix. For an (*n, k*) cyclic code, the generator polynomial $g(X)$ has degree n-k and is given as

$$g(X) = g_0 + g_1 X + ... + g_{n-k} X^{n-k} \tag{4.15}$$

Where g_i in binary (0 or 1) and g_0 = g_{n-k} =1. The k-bit information is also written in polynomial form as

$$u(X) = u_0 + u_1 X + ... + u_{k-1} X^{k-1} \tag{4.16}$$

The codeword for a given k-bit information sequence is obtained from the polynomial coefficients of the generator polynomial multiplied by the message polynomial, thus the codeword is obtained as

$$c(X) = u(X) g(X) = c_0 + c_1 X + ... + c_{n-1} X^{n-1} \tag{4.17}$$

It is important to note that a codeword described by a polynomial $c(X)$ is a valid codeword for a cyclic code when generator polynomial $g(X)$ divides $c(X)$ with no remainder. In linear block codes, we have the first k codeword symbols equal to information bits and the remaining codeword symbols equal to parity bits. In a similar way, a cyclic code can be put in systematic form by first multiplying the message polynomial $u(X)$ by X^{n-k}, we get

$$X^{n-k}u(X) = u_0 X^{n-k} + u_1 X^{n-k+1} + ... + u_{k-1} X^{n-1} \tag{4.18}$$

By this the message bits get shifted to the k rightmost digits. Dividing this equation by $g(X)$, we get

$$\frac{X^{n-k}u(X)}{g(X)} = q(X) + \frac{p(X)}{g(X)} \tag{4.19}$$

Where $q(X)$ is a polynomial of degree at most $k-1$ and $p(X)$ is a remainder polynomial of degree at most $n-k-1$. By multiplying with $g(X)$ we get

$$X^{n-k}u(X) = q(X)g(X) + p(X) \tag{4.20}$$

$$\text{or} \quad p(X) + X^{n-k}u(X) = q(X)g(X) \tag{4.21}$$

Thus, $p(X) + X^{n-k}u(X)$ is a valid code word.

Because this is divisible by $g(X)$ with no remainder. Thus by substituting the values we get valid cyclic codeword as

$$\begin{aligned} & p(X) + X^{n-k}u(X) \\ & = p_0 + p_1 X + ... + p_{n-k-1} X^{n-k-1} + u_0 X^{n-k} + u_1 X^{n-k-1} + ... + u_{k-1} X^{n-1} \end{aligned} \tag{4.22}$$

The codeword for this polynomial has the first k bits as message bits, and last n-k bits as parity bits.

4.2.5 Interleaving Technique:

Interleaving technique given in Proakis and Salehi (2014) is frequently used in digital communication to improve the performance of forward error correcting codes. In many communication channels errors typically occur in bursts rather than independently. If the number of errors within a code word exceeds the error-correcting code's capability, it fails to recover the original code word. Interleaving arrests this problem by shuffling source symbols across several code words, thereby creating a more uniform distribution of errors. Therefore, interleaving is widely used for burst error-correction. The interleaving technique improves performance of coding in fading channels by transforming bursty channel to a channel having independent errors. Thus coding is typically combined with interleaving to mitigate the effect of error bursts. A block diagram of a system using interleaving is shown in fig. 4.10.

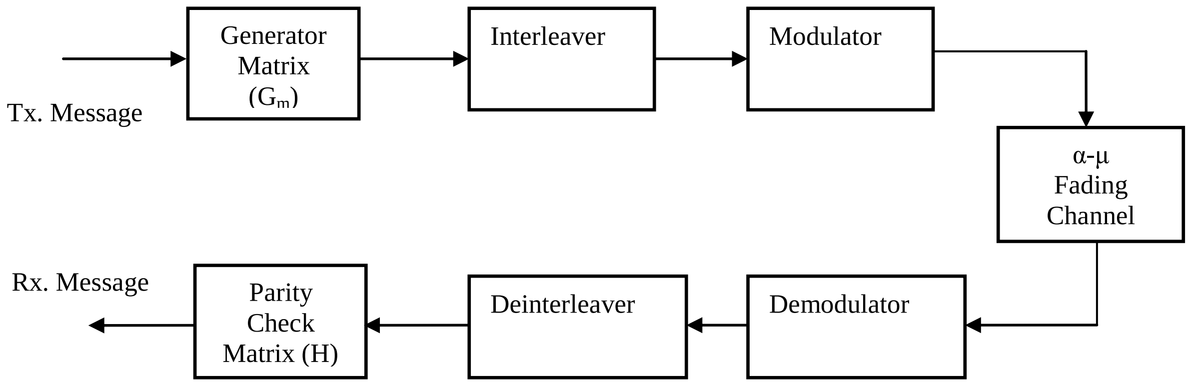

Fig. 4.10. Block diagram of system using interleaving for burst error channel

Unlike wireline AWGN, in wireless due to multipath and fading, channel may exhibits bursty error characteristic, that is signal fading due to time-invariant multipath propagation often causes the signal to fall below the noise level, resulting in large number of errors i.e. burst errors. Such error clusters are not usually corrected by codes designed for statistically independent errors. Suppose a burst of errors of length b has occurred which means a sequence of b-bit in errors. Then, the burst error correction capability of a systematic (n, k) code, which has $(n - k)$ parity check bits, is $b < [½(n - k)]$.

4.2.5.1 Random Interleaving:

The encoded data are processed by the interleaver and transmitted over the channel. At the receiver, the deinterleaver puts the data in proper sequence and passes them to the decoder. As a result of the interleaving/deinterleaving, error bursts are spread out in time so that errors within a code word appear to be independent. In a random interleaver, a block of N input bits are written in the interleaver in the same order in which they are received. Then, they are read out in a random manner. A random interleaver is a random permutation π. The interleaver has a corresponding de-interleaver (π^{-1}) that acts on the interleaved data sequence and restores it to the original order.

If the input data sequence is $U = [u_1, u_2, \ldots, u_N]$, then permuted data sequence is $U \times P$, where P being the interleaving matrix with single 1, which is randomly located in each row and column, all other entries being zero. The de-interleaving matrix (P^T) is transpose of the interleaving matrix (P).

4.2.5.2 Block Interleaving:

In block interleaver, the encoded data are recorded by the interleaver and transmitted over the channel. The interleaver formats the encoded data in a rectangular array of *m* rows and *n* columns. The bits are read out columnwise and transmitted over the channel. At the receiver, the deinterleaver stores the data in the same rectangular array format, but they are read out rowwise, one codeword at a time. As a result of this reordering of the data during transmission, a burst of errors of length $l = m \times b$ is broken up into *m* bursts of length *b*. Thus an *(n, k)* code that can handle burst errors of length $b < [\frac{1}{2}(n - k)]$ can be combined with an interleaver of degree *m* to create an interleaved *(mn, mk)* block code that can handle burst error of length *mb*.

4.2.5.3 Convolutional Interleaving:

In block coding the interleaver spreads errors across different codewords. In convolutional codes there is no concept of codeword, hence a different type of interleaving called 'convolutional interleaving' is required to mitigate the effect of burst errors. In this the encoder output is multiplexed into buffer of increasing size, from no buffering to a buffer of size (N-1). The channel input is multiplexed from these buffers into the channel. The reverse operation is performed by decoder. Thus the convolutional interleaver takes sequential outputs of the encoder and separates them by N-1 other symbols in the channel transmission, thereby breaking up burst errors in the channel.

4.2.6 Performance Analysis of Coded α-μ Channel:

BER performance for block coding, cyclic coding, and interleaving techniques over α-μ fading channel is obtained by Monte-Carlo simulation. Results for different α and μ values are shown in fig.4.11 to fig.4.18. In this simulation 500000 samples have been considered for a particular α and μ combination. The codeword n = 7, and each message is having binary sequence of length k = 4. The following generator matrix has been used in block coding simulation

$$G_m = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \qquad (4.23)$$

The generator polynomial considered in the simulation cyclic coding is

$$g_p = \begin{bmatrix} 1 & 0 & 1 & 1 \end{bmatrix} \qquad (4.24)$$

Communication system shown in fig.4.10 is considered in the simulation for performance analysis. In fig.4.11 the BER performance for α=7/4, μ=2 is shown. It is seen that BER performance for a given SNR improves after coding and further improves after interleaving. At 5dB SNR, the uncoded BER is 0.2×10^{-2}, it improves after block or cyclic coding to 4.5×10^{-3}, and further after interleaving in both the coding it is nearly 10^{-3}. In fig.4.12 the BER performance for α=1, μ=1 is plotted. Here, it is observed, at 10dB SNR, the uncoded BER is 6.0×10^{-2}, it improves after block or cyclic coding to 5.0×10^{-2}, and further after interleaving in both the coding it is nearly 2.5×10^{-2}.

Fig.4.13 shows the BER performance for α=1 and μ=3, it is found that for 0dB SNR, the uncoded BER is 9.0×10^{-3}, it improves after block or cyclic coding to 4.0×10^{-3}, and further after interleaving in both the coding it is 6.0×10^{-4}. Similar, interpretation can be made for fig.4.14, for α=1, μ=7, but since μ has increased, as anticipated improvement in BER can be observed than fig. 4.13. Further, in fig.4.15 the BER performance for α=3, μ=1 is given. It is observed, at 10dB SNR, the uncoded BER is 1.2×10^{-2}, it improves after block or cyclic coding to 4.5×10^{-3}, and further after interleaving in both the coding it is nearly 1.4×10^{-3}. Similarly it is found that in fig.4.16, for α=7, μ=1, now since α has increased, there is again improvement in BER observed than fig. 4.15.

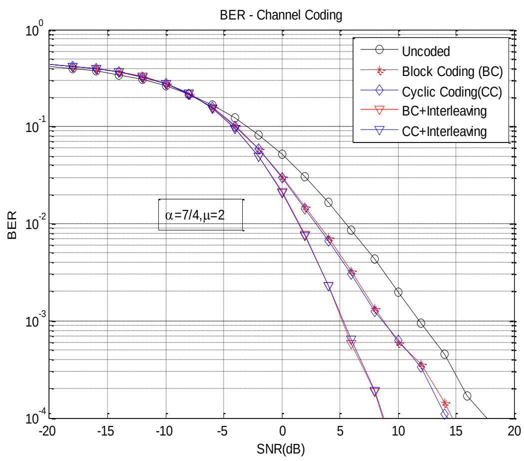

Fig. 4.11. BER after coding over α-μ fading for α=7/4, μ=2

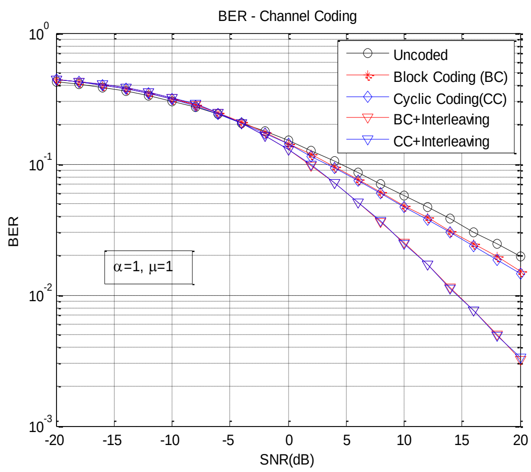

Fig. 4.12. BER after coding over α-μ fading for α=1, μ=1

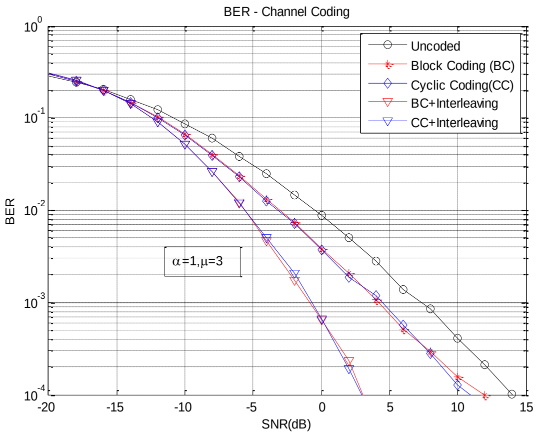

Fig. 4.13. BER after coding over α-μ fading for α=1, μ=3

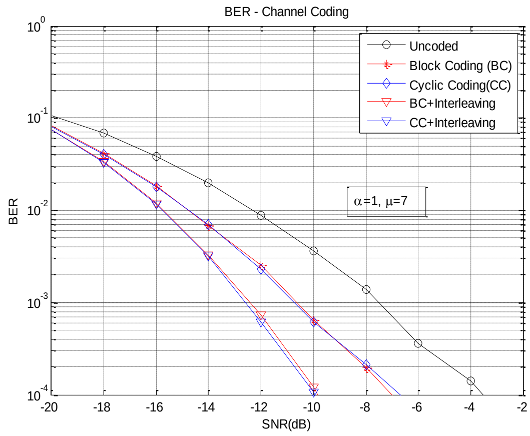

Fig. 4.14. BER after coding over α-μ fading for α=1, μ=7

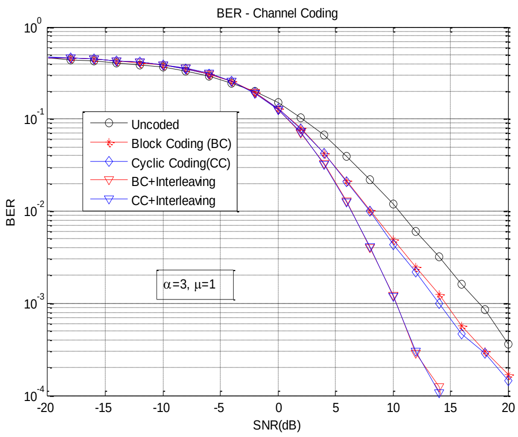

Fig. 4.15. BER after coding over α-μ fading for α=3, μ=1

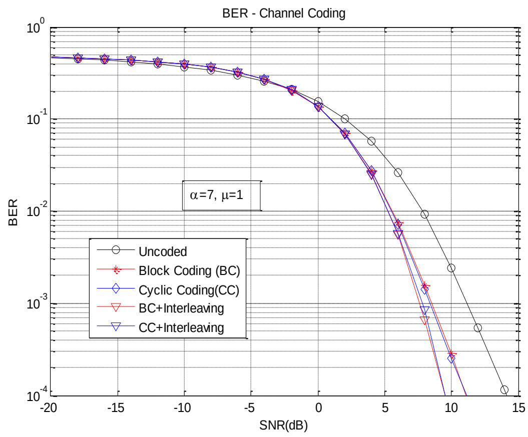

Fig. 4.16. BER after coding over α-μ fading for α=7, μ=1

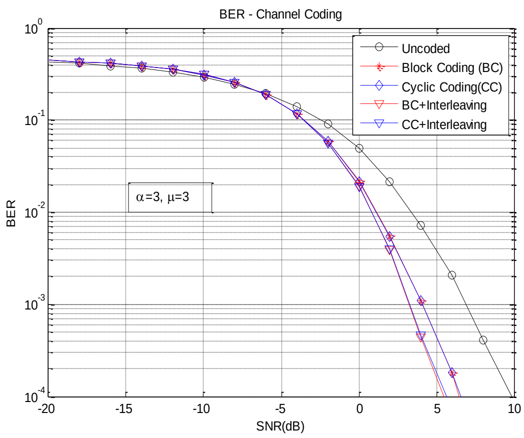

Fig. 4.17. BER after coding over α-μ fading for α=3, μ=3

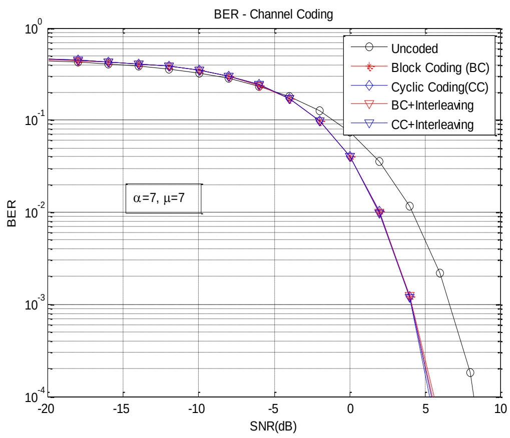

Fig. 4.18. BER after coding over α-μ fading for α=7, μ=7

In fig.4.17 the BER performance for α=3 and μ=3 is shown. It is found that for 5dB SNR, the uncoded BER is 3.5×10^{-3}, it improves after block or cyclic coding to 4.0×10^{-4}, and further after interleaving in both the coding it is 1.7×10^{-4}. Also, in fig.4.18 the BER performance for α=7, μ=7 is given, where it is observed, at 5dB SNR, the uncoded BER is 5.0×10^{-3}, it improves after block or cyclic coding and interleaving to nearly 2.0×10^{-4}. From the simulation results of fig.4.11 and fig.4.18, we find that at higher value of α and μ, the BER performance has significantly improved, due to high power and large number of multipath clusters.

In this analysis, the simulated results of BER performance over α-μ fading for block coding, cyclic coding and interleaving schemes have been illustrated. The effect of α and μ parameters on BER performance is brought out. The result shown here have been published in research paper titled ***"Bit Error Rate Analysis of Block Coded Wireless System over α-μ Fading Channel***", in International Journal of Advance Research in Computer and Communication Engineering (IJARCCE), ISSN 2378-1021, vol.4, iss.8, Aug 2015, pp.165-169. (*DOI 10.17148/IJARCCE.2015.4834*).

4.3 Performance of CDMA system over α-μ Fading Channel

4.3.1 Introduction to CDMA:

Code division for multiple access (CDMA) is a multiple access technology. In multiuser systems there are multiple users who share the common radio channel. This mechanism is termed 'Multiple Access' as technology. Initially in first generation (1G) cellular system, frequency division multiple access (FDMA) technology were used, where different users were allocated different frequency band. In FDMA channel bandwidth is subdivided into frequency non-overlapping subchannels. This is commonly used in wireline channels to accommodate multiple users for voice and data transmission. In second generation (2G) systems such as GSM, time division multiple access (TDMA) technology were used, where different users were allocated different time slots. In TDMA frame duration is sub-divided in non-overlapping sub-intervals. Each user transmits information in the assigned particular time slot in each frame. In speech signals containing long pauses, both the FDMA and TDMA tend to be inefficient because certain amount of frequency slot or time slot is not utilized. This problem is avoided, by assigning a unique code sequence to each user pair to spread the information signal across the assigned frequency band. The information signal is retrieved at receiver by correlation of received signal with the same assigned code for that user pair. This multiple access system is called as CDMA or spread spectrum multiple access (SSMA). Bamisaye Ayodeji J., et. al., (2010), evaluated downlink performance of a multiple-cell, rake receiver assisted CDMA mobile system. The 3G systems such as WCDMA, HSDPA/HSUPA, CDMA 2000 are based on CDMA, where different users are allocated different codes.

Let us consider an example of two user CDMA system:

(i) user 0, symbol a_0, code C_0

(ii) user 1, symbol a_1, code C_1

$$C_0 = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \text{ and } C_1 = \begin{bmatrix} 1 & -1 & -1 & 1 \end{bmatrix} \tag{4.25}$$

Multiply symbol of each user with respective code

$$a_0 \times C_0 = a_0 \times \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} a_0 & a_0 & a_0 & a_0 \end{bmatrix} \tag{4.26}$$

$$a_1 \times C_1 = a_1 \times \begin{bmatrix} 1 & -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} a_1 & -a_1 & -a_1 & a_1 \end{bmatrix} \tag{4.27}$$

Both the signals are transmitted over same wireless channel. The combined signal is generated as the sum of signals of both users.

$$\text{Combined Signal} = a_0 C_0 + a_1 C_1 = \begin{bmatrix} a_0 & a_0 & a_0 & a_0 \end{bmatrix} + \begin{bmatrix} a_1 & -a_1 & -a_1 & a_1 \end{bmatrix} \tag{4.28}$$

$$\text{Combined Signal} = \begin{bmatrix} a_0 + a_1 & a_0 - a_1 & a_0 - a_1 & a_0 + a_1 \end{bmatrix} \tag{4.29}$$

Each user has to extract his corresponding signal from the received combined signal, by correlating the received combined signal with its corresponding code. At receiver of user 0, the correlation between the combined signal and code C_0 will be carried out as

$$\text{Correlated Signal} = \left[1\times(a_0+a_1) \quad 1\times(a_0-a_1) \quad 1\times(a_0-a_1) \quad 1\times(a_0+a_1)\right] = 4\times a_0 \quad (4.30)$$

Thus user 0, can extract its symbol a_0 from transmitted CDMA. Similarly user 1, can extract its symbol a_1. As seen in eq.(4.26) and (4.27), each symbol generates 4 chips, i.e. N = 4 chips per symbol. If we consider symbol rate as 1kbps = 1000 symbols per second.

Then time for one symbol = 1/1000 sec = 1ms. Symbol time (T_s) = 1 ms

Bandwidth required for this system, BW = 1 / T_s = 1 KHz

In CDMA , during one symbol time, 4 chips will be transmitted.

Chip time (T_c) = ¼ ms = 0.25 ms

Thus BW of CDMA system, BW = 1/T_c = 1/0.25 = 4 KHz

Therefore, the BW(or spectrum) of the original signal is spread in CDMA system. Hence, the CDMA system is also known as a “Spread Spectrum” system.

BW of CDMA = N × (BW of original signal), where N = code length = T_s / T_c.

Thus, in CDMA BW or spectrum is spread by a factor of N. The CDMA codes are also known as “Spreading code”, and length of code N is called “Spreading factor”. The CDMA codes are orthogonal codes, because correlation between codes is zero.

$$\text{Correlation}\left[C_0, C_1\right] = 1\times1 + 1\times(\text{-}1) + 1\times(\text{-}1) + 1\times1 = 1+(-1)+(-1)+1 = 0 \qquad (4.31)$$

There can be 4 orthogonal codes from the length of code (N=4).

$$C_0 = [1 \quad 1 \quad 1 \quad 1], \quad C_1 = [1 \quad -1 \quad -1 \quad 1],$$
$$C_2 = [1 \quad -1 \quad 1 \quad -1], \quad C_3 = [1 \quad 1 \quad -1 \quad -1] \tag{4.32}$$

Any two codes are orthogonal codes. Thus, the number of orthogonal codes corresponding to length *N* is *N*. Thus the number of users that can access the common channel is *N*. The spreading codes are also known as Pseudo Noise (PN) sequence, because they look like a random sequence of +1 and -1, but actually they are not random. One technique to generate spreading codes of longer length is through, Linear Feedback Shift Register (LFSR).

Let in a multi-user CDMA, $C_0(n)$ be code sequence of user 0 and $C_1(n)$ be code sequence of user 1. Symbol corresponding to user 0 be a_0 and that for user1 be a_1.

$$\text{Transmitted CDMA Signal, } x(n) = a_0 C_0(n) + a_1 C_1(n) \tag{4.33}$$

$$\text{Received Signal, } y(n) = h \times [a_0 C_0(n) + a_1 C_1(n)] + w(n) \tag{4.34}$$

Correlation at receiver of user0;

$$r_0 = \frac{1}{N} \sum_n y(n) \times C_0(n) \tag{4.35}$$

$$r_0 = \frac{1}{N} \sum_n \{h[a_0 C_0(n) + a_1 C_1(n)] + w(n)\} \times C_0(n) \tag{4.36}$$

$$r_0 = \frac{1}{N} \sum_n h a_0 C_0{}^2(n) + \quad \frac{1}{N} \sum_n h a_1 C_1(n) C_0(n) + \frac{1}{N} \sum_n w(n) C_0 n) \tag{4.37}$$

Eq. (4.37) has three parts given as in eq. (4.38)

$$r_0 = \text{Desired signal part + Interference (Multi-user Interferer, MUI) + Noise} \tag{4.38}$$

Let us find power in each part separately,

(i) Desired Signal Part:

$$\frac{1}{N} \sum_n h a_0 C_0{}^2(n) = h\ a_0 \frac{1}{N} \sum_n C_0{}^2(n) = \frac{h a_0}{N} N = h a_0 \tag{4.39}$$

Autocorrelation of PN sequences for user 0,

$$r_{0,0}(0) = \frac{1}{N}\sum_n C_0{}^2(n) = \frac{1}{N}\sum_n 1 = \frac{N}{N} = 1 \tag{4.40}$$

Since $C_0(n)$ can be either $+1$ or -1. Hence $C_0{}^2(n)$ will be $+1$.
Since $r_{00}(0) = 1$, therefore, power $E\{r_{00}{}^2(0)\}$ will also be one.

$$\text{Power of Desired signal} = |h|^2\,|a_0|^2 = |h|^2\,P_0 \tag{4.41}$$

(ii) Interference by Multi-user Interferer(MUI):

$$\frac{1}{N}\sum_n h a_1 C_1(n) C_0(n) = h a_1 \frac{1}{N}\sum_n C_1(n) C_0(n) \tag{4.42}$$

$$\text{Here cross correlation, } r_{01}(k) = \frac{1}{N}\sum_n C_0(n) C_1(n+k) \tag{4.43}$$

$$\text{Power } = E\left\{r_{01}{}^2(k)\right\} = \frac{1}{N^2} E\left\{\sum_m\sum_n C_0(m)C_1(m+k)C_0(n)C_1(n+k)\right\} \tag{4.44}$$

$$\text{or } E\left\{r_{01}{}^2(k)\right\} = \frac{1}{N^2}\sum_n E\left\{C_0{}^2(n)\right\}\times E\left\{C_1{}^2(n+k)\right\} \tag{4.45}$$

$$\text{or } E\left\{r_{01}{}^2(k)\right\} = \frac{1}{N^2}\sum_n 1\times 1 = \frac{N}{N^2} = \frac{1}{\mathrm{N}} \tag{4.46}$$

Power of MUI can be given from eq.(4.42) and (4.46)

$$\text{Power of MUI} = |h|^2\,|a_1|^2\,\frac{1}{N} = \frac{|h|^2 P_1}{N} \tag{4.47}$$

(iii) Noise Part:

$$\text{Noise part} = \frac{1}{N}\sum_n \omega(n) C_0(n) \tag{4.48}$$

$$\text{Noise power} = \left[\frac{1}{N}\sum_m \omega(m) C_0(m)\right]\left[\frac{1}{N}\sum_n \omega(n) C_0(n)\right] \tag{4.49}$$

$$\text{Noise power} = \frac{1}{N^2}\sum_m\sum_n E\left[\omega(m)\omega(n)C_0(m)C_0(n)\right] \tag{4.50}$$

When $m \neq n$, all will be zero. For $m = n$,

$$\text{Noise power} = \frac{1}{N^2}\sum_m E\left[C_0^{\,2}(m)\right]\ \mathrm{E}\left[\omega^2(m)\right] \tag{4.51}$$

$$\text{Here } E\left[C_0^{\,2}(m)\right] = 1 \text{ and } E\left[\omega^2(m)\right] = \sigma_\omega^{\,2} \tag{4.52}$$

$$\text{Hence, Noise power} = \frac{1}{N^2}\sigma_\omega^{\,2}\sum_m 1 = \frac{N\sigma_\omega^{\,2}}{N^2} = \frac{\sigma_\omega^{\,2}}{N} \tag{4.53}$$

Now using eq. (4.41), (4.47) and (4.53), signal to (interference +noise) ratio (SINR) can be found as

$$\text{Therefore, SINR} = \frac{\text{Signal Power}}{\text{MUI Power + Noise Power}} = \frac{|h|^2 P_0}{\frac{|h|^2 P_1}{N} + \frac{\sigma_\omega^{\,2}}{N}} \tag{4.54}$$

$$\text{SINR} = \frac{|h|^2 P_0}{|h|^2 P_1 + \sigma_\omega^{\,2}} \times \mathrm{N} \tag{4.55}$$

4.3.2 Multi-User CDMA System:

CDMA system is also known as a “Spread Spectrum” system since the bandwidth or spectrum of the original signal is spread in this system. In CDMA, spectrum is spread by a factor of *N*, where *N* is code length for each symbol. The CDMA codes are known as “Spreading code”, and length of code *N* is called “Spreading factor”. Any two codes in CDMA systems are orthogonal codes. Thus, the number of orthogonal codes corresponding to length *N* is *N*. Hence, the number of users that can access the common channel is *N*. The spreading codes are also known as pseudo noise (PN) sequence, because they look like a random sequence of +1 and -1, but actually they are not random. One technique to generate spreading codes of longer length is through, linear feedback shift register (LFSR).

Let in a multi-user CDMA, $C_i(n)$ be code sequence and a_i be symbol corresponding to i^{th} user.

$$\text{Transmitted CDMA Signal, } x(n) = \sum_{i=0}^{k-1} a_i C_i(n) \tag{4.56}$$

$$\text{Received Signal, } y(n) = h \times \left(\sum_{i=0}^{k-1} a_i C_i(n) \right) + w(n) \tag{4.57}$$

At receiver signal of each user is retrieved by correlation with code of the respective user. Accordingly for j^{th} user, correlated output will be;

$$r_j = \frac{1}{N} \sum_n y(n) \times C_j(n) \tag{4.58}$$

$$r_j = \frac{1}{N} \sum_n \left(h \times \left(\sum_{i=0}^{k-1} a_i C_i(n) \right) + w(n) \right) \times C_j(n) \tag{4.59}$$

The multiplication of C_i and C_j on right hand side will have one term $i = j$ for desired user, terms corresponding to $i \neq j$ are undesired users causing multi-user interference (MUI), and a noise term regarding $w(n)$. Calculating the power for all the terms. The term $i = j$, for desired user

$$\begin{aligned} \text{Desired signal} &= \frac{1}{N} \sum_n h \times \left(a_j C_j(n) \right) C_j(n) \\ &= h a_j \frac{1}{N} \sum_n C_j{}^2(n) = h a_j \end{aligned} \tag{4.60}$$

$$\text{Since } C_j(n) \text{ is +1 or -1, hence } \frac{1}{N} \sum_n C_j{}^2(n) = \frac{1}{N} \sum_n 1 = \frac{N}{N} = 1$$

$$\text{Power of desired } j^{th} \text{ user } = |h|^2 \left| a_j \right|^2 = |h|^2 P_j \tag{4.61}$$

There are total k users, and i^{th} user may be from i=0 to k-1. Out of these k users, j^{th} user is desired user. Hence, there will be k-1 undesired users causing MUI for the desired user i.e. j^{th} user. Let us evaluate MUI caused by these k-1 undesired users to j^{th} desired user. For this, first we will evaluate MUI by one of the undesired user, say (i=0), out of k-1 undesired users.

Here mean of cross correlation, r_{0j} will be

$$E\left\{ r_{0j} \right\} = E\left\{ \frac{1}{N} \sum_n C_0(n) C_j(n) \right\} = \frac{1}{N} \sum_n E\{C_0(n) C_1(n)\} = 0 \tag{4.62}$$

And power of cross correlation, r_{0j} will be

$$E\left\{\left|r_{0j}\right|^2\right\} = E\left\{\frac{1}{N}\sum_n C_0(n)C_j(n) \times \frac{1}{N}\sum_m C_0(m)C_j(m)\right\}$$

$$= E\{\frac{1}{N^2}\sum_n\sum_m C_0(n)C_j(n)C_0(m)C_j(m)\}$$

$$= \frac{1}{N^2}\sum_n\sum_m E\{C_0(n)C_0(m)\} \times E\{C_j(n)C_j(m)\}$$

$$= \frac{1}{N^2}\sum_n E\{C_0{}^2(n)\} \times E\{C_j{}^2(n)\} = \frac{1}{N^2}\sum_n 1\times 1 = \frac{N}{N^2} = \frac{1}{N} \tag{4.63}$$

Since, terms of $m \neq n$ will be zero because codes are orthogonal. Now, the MUI by the undesired user (i=0), out of k-1 undesired users, for j^{th} desired user, may be given with help of eq.(4.59) and (4.63).

MUI by (i=0) undesired user

$$=E\left\{\left|h\right|^2 \times \left|a_0\right|^2 \times \left|r_{0j}\right|^2\right\} = \left|h\right|^2 P_0 \ \frac{1}{N} \tag{4.64}$$

Therefore MUI by all the k-1 undesired users will be

$$= \left|h\right|^2 \frac{1}{N} \sum_{i=0, i\neq j}^{k\text{-}1} P_i \tag{4.65}$$

Finally the Noise power, $\frac{1}{N^2}\sum_n E\left[\omega^2(n)\right]E\left[C_j{}^2(n)\right]$

$$= \frac{1}{N^2}\sigma_\omega{}^2\sum_n 1 = \frac{N\sigma_\omega{}^2}{N^2} = \frac{\sigma_\omega{}^2}{N} \tag{4.66}$$

Therefore, signal to (interference +noise) ratio, SINR for j^{th} user will be

$$\text{SINR } (\gamma_j) = \frac{\left|h\right|^2 P_j}{\left|h\right|^2\left[\sum_{i=0,\ i\neq j}^{k-1} P_i\right] + \sigma_\omega{}^2} \times N \tag{4.67}$$

4.3.3 Walsh –Hadamard Code:

Walsh-Hadamard code defined by Proakis and Salehi (2014), having length $N = T_s/T_c$ and synchronized in time, are orthogonal for a symbol. Thus the cross correlation of any two codes is zero. Since synchronizing of users is possible on downlink, because of all signals originating from same transmitter, but synchronizing is challenging in uplink case. Hence this code is used for downlink channels. Walsh-Hadamard sequence of length N is obtained from the rows of an $N \times N$ Hadamard matrix H_N. For N=2 the Hadamard matrix is given as

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{4.68}$$

Larger Hadamard matrics with help of H_2 and so on can be obtained from eq.(4.69)

$$H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & -H_N \end{bmatrix} \tag{4.69}$$

Each row of H_N provides different chip sequence, thus the number of spreading code in Walsh-Hadamard code is N. The direct sequence spread spectrum (DSSS) with Walsh-Hadamard codes can support maximum $N = T_s / T_c$ users. Substituting N=2 in eq. (4.69), we get codes for 4 users, as shown in each row of eq.(4.70).

$$H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \tag{4.70}$$

4.3.4 Performance Analysis of Multi-user CDMA system over α-μ Channel:

BER performance for multi-user CDMA system over α-μ fading channel is obtained by Monte-Carlo simulation. The CDMA communication system shown in fig.4.19 has been considered in this simulation. BER performance simulated results for different α and μ values are shown in fig. 4.20 to fig.4.23. In these simulation 100000 bits have been considered for a particular α and μ combination. The codeword n = 7, and each message is having binary sequence of length k = 4, and is BPSK modulated. The α-μ fading channel is assumed to be stationary for each codeword and changes independently for different codewords. The generator matrix given in eq. (4.71) is used by encoder for generating codewords.

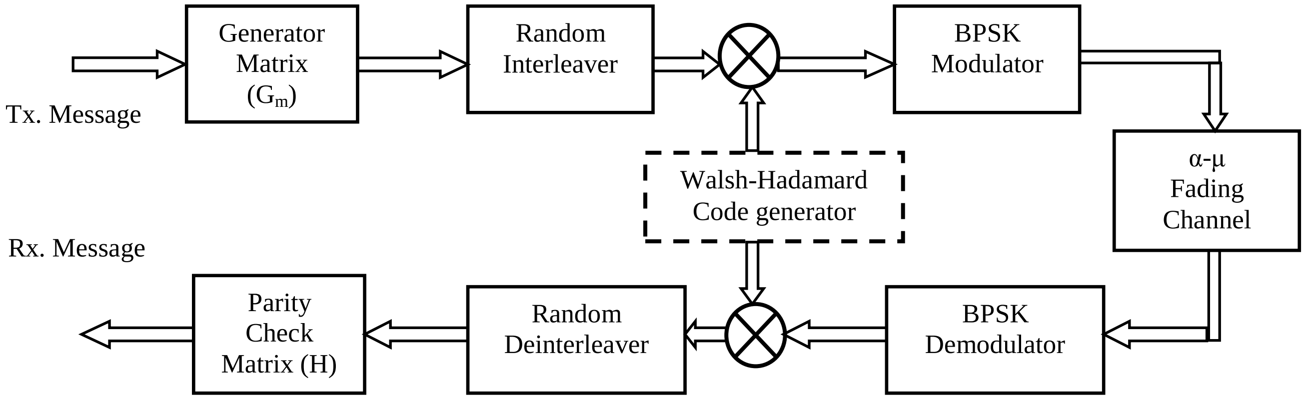

Fig. 4.19. Multi-user CDMA System model considered for simulation

$$G_m = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{4.71}$$

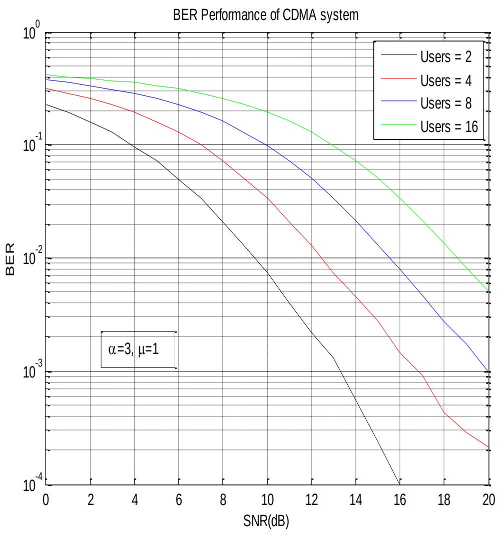

Fig. 4.20. BER of CDMA system over α-μ fading for α=3, μ=1

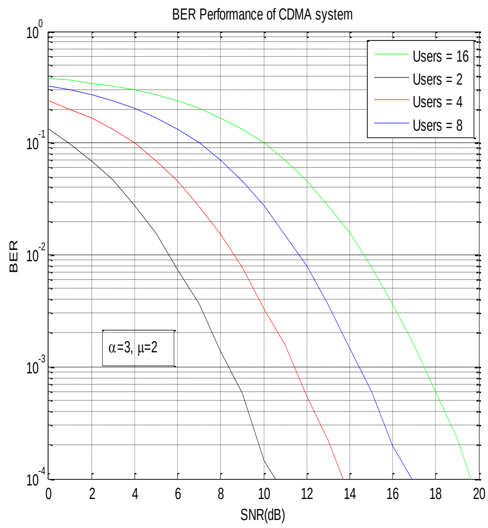

Fig. 4.21. BER of CDMA system over α-μ fading for α=3, μ=2

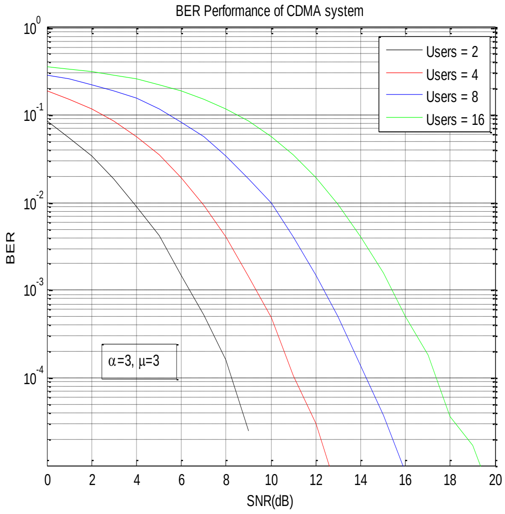

Fig. 4.22. BER of CDMA system over α-μ fading for α=3, μ=3

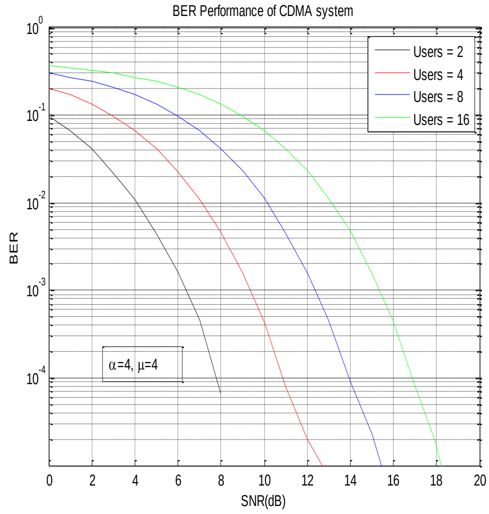

Fig. 4.23. BER of CDMA system over α-μ fading for α=4, μ=4

The Walsh-Hadamard code H_2, H_4, H_8, and H_{16} for 2, 4, 8, and 16 users respectively is obtained from eq. (4.69). Energy requirement for different users data for transmission will increase, as we increase the value of N. Therefore, for faithful comparison we have normalized the user data from the value of N. In this paper we have used second order Walsh-Hadamard matrix for transmission of two user data, fourth order matrix for transmission of four user data and so on. This scheme will utilize minimum spectrum when users are less.

Fig.4.20 to 4.23 shows the BER performance of multi-user CDMA system over α-μ fading channel for different values of α and μ. In fig. 4.20, for α=3, μ=1, the BER performance at 10dB SNR for 2, 4, 8, and 16 users are observed as 7.0×10^{-3}, 3.3×10^{-2}, 9.0×10^{-2}, and 2.0×10^{-1} respectively. It is seen that when number of users increases the BER also increases. It is because with increase in users, the multi-user interference also increases. As a result the SNR decreases, hence the BER increases.

It is observed in fig. 4.21, for α=3, μ=2, the BER performance at 10dB SNR for 2, 4, 8, and 16 users are 1.5×10^{-4}, 3.0×10^{-3}, 2.8×10^{-2}, and 10^{-1} respectively. BER is better in fig. 4.21 than fig. 4.20, because multipath clusters μ has increased from 1 to 2. It is further seen from fig. 4.22, for α=3, μ=3, after increasing the μ, the BER performance improvements at 10dB SNR for 4, 8, and 16 users are 4.5×10^{-4}, 9.0×10^{-3}, and 5.5×10^{-2} respectively. We also notice that the BER performance improvements in fig. 4.23 for α=4 and μ=4, at 10dB SNR for 4, 8, and 16 users are 3.5×10^{-4}, 10^{-2}, and 6.5×10^{-2} respectively, which has slight improvements than fig. 4.22. An inference can be drawn from this result that the BER performance saturates at higher value of α and μ.

From the simulation results of fig.4.20 to Fig.4.23, we find that at higher value of α and μ, the BER performance has significantly improved, due to high power α and large number of multipath clusters μ. It is also noticed that for larger number of users the BER performance decreases due to increase in multi user interference (MUI).

In this analysis α-μ fading channel has been briefly introduced. BER performance of single carrier multi-user CDMA system, with random interleaving are analysed. The effect of α and μ parameters on BER performance is brought out. It has been analyzed that increase in number of users adversely affects the error performance, due to increase in multi user interference. The results have been documented in a form of research paper titled, ***"Error Analysis of Multi-User CDMA System over α-μ Fading Channel***", and is published in International Journal of Advance Research in Computer and Communication Engineering (IJARCCE), ISSN 2378-1021, vol.4, iss.10, Oct 2015, pp.533-537. (*DOI 10.17148/IJARCCE.2015.410121*).

CHAPTER 5

PERFORMANCE OF MULTI-ANTENNA *α-μ* CHANNEL

5.1 Performance of Multi-Antenna *α-μ* Channel:

With receiver diversity technique, multiple copies of the same signal is received on different branches, which undergo independent fading. The impact of antenna diversity on the capacity of wireless systems is reported by Winters, et.al. (1994). Aldalgamouni Taimour, et. al., (2013) presented performance of selected diversity techniques over the α-μ fading channels. Wang and Beaulieu (2009a) described switching rate of two-branch selection diversity. Signals received through these independent fading paths are combined to obtain a signal that is then passed through the demodulator. If one branch undergoes a deep fade, the another branch may have strong signal. In space diversity fading are minimized by the simultaneous use of two or more physically separated antennas. Thus having more than one path to select the SNR at receiver may be improved by selecting appropriate combining technique. The SNR γ is a random variable and is given for different diversity schemes.

$$\gamma \in \{\gamma_{SC}, \gamma_{MRC}, \gamma_{EGC}, \gamma_{SSC}\} \tag{5.1}$$

Combining of independent fading path signals, to mitigate fading, can be done in different ways, which vary in complexity and performance. Following diversity combining techniques for *α-μ* fading channel are discussed.

5.1.1 Selection combining:

Selection combining as given by Rappaport (2011) is based on the principle of selecting the best signal among all the signals received from different branches at the receiving end.

In this method, the receiver monitors the SNR of the incoming signal using switch logic. In fig.5.1 there are M independent α-μ fading channels providing M diversity branches. SNR of each branch is monitored simultaneously. The branch with highest instantaneous SNR is connected to demodulator. Thus, the output SNR of combiner is equal to maximum SNR of all branches. Since only one branch output is taken, co-phasing is not required.

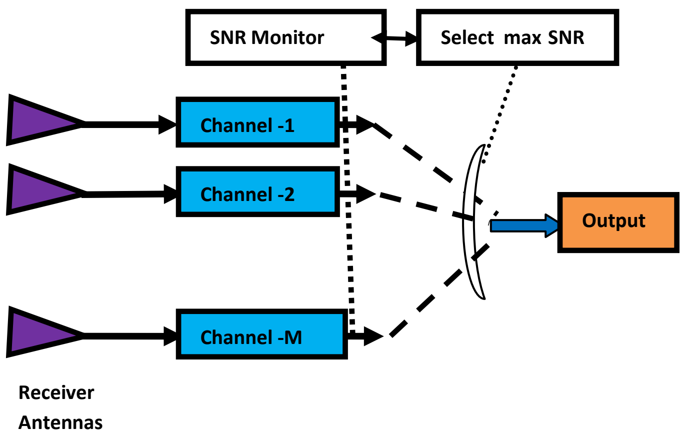

Fig.5.1. Selection combining

Selection combining is based on the principle of selecting the best signal among all the signals received from different branches at the receiving end. In this method, the receiver monitors the SNR of the incoming signal using switch logic. The branch with highest instantaneous SNR is connected to demodulator. SNR of selection combining is given as

$$\gamma_{SC} = \text{Max}\left(R_1^2, R_2^2\right) \times SNR \tag{5.2}$$

Where R_1 and R_2 represent the fading envelope for two diversity paths seen by two different antennas. R_1^{2} is fading power between transmitter and 1st receiving antenna. R_2^{2} is fading power between transmitter and 2nd receiving antenna. SNR is unfaded signal to noise ratio. PDF of selection combining signal to noise ratio (γ) can be obtained by differentiating the CDF of the α-μ fading distribution obtained in eq.(3.62) as given below:

$$f_{\gamma SC}(\gamma) = \frac{d}{d\gamma}\, P(\gamma) \tag{5.3}$$

$$f_{\gamma SC}(\gamma) \quad = \quad \frac{\mathrm{d}}{\mathrm{d}\gamma}\left[\frac{\gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(u)}\right]^{L} \tag{5.4}$$

$$f_{\gamma SC}(\gamma) \quad = \quad \mathrm{L}\left[\frac{\gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(u)}\right]^{L-1} \quad \frac{d}{d\gamma}\left[\frac{\gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(u)}\right] \tag{5.5}$$

Let us substitute the following definition of $\gamma(s, x)$ i.e. Lower Incomplete Gamma function in eq. (5.5)

$$\gamma(s, x) = \int_{0}^{x} \mathrm{t}^{\mathrm{s-1}}\, e^{-t}\, \mathrm{dt}$$

We get, $$f_{\gamma SC}(\gamma) \; = \; \mathrm{L}\left[\frac{\gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(u)}\right]^{L-1} \quad \frac{d}{d\gamma}\left[\frac{\int_{0}^{\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} t^{\mu-1} e^{-t} dt}{\Gamma(u)}\right] \tag{5.6}$$

For solving the integration of eq.(5.6), by using eq. (0.410) of Gradshteyn and Ryzhik (2007) given as

$$\frac{d}{da}\int_{\psi(a)}^{\varphi(a)} f(x,a)dx = f\left(\varphi(a), a\right)\frac{d\varphi(a)}{da} - f\left(\psi(a), a\right)\frac{d\psi(a)}{da} + \int_{\psi(a)}^{\varphi(a)} \frac{d}{da}\, f(x,a)dx$$

$$f_{\gamma SC}(\gamma) = \mathrm{L}\left[\frac{\gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(u)}\right]^{L-1} \quad \frac{1}{\Gamma(\mathrm{u})}\left[\left(t^{\mu-1}e^{-t}\right)\frac{d}{d\gamma}\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2} - 0 + \int_{0}^{\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} \frac{d}{d\gamma}\left(t^{\mu-1}e^{-t}\right)dt\right] \tag{5.7}$$

$$f_{\gamma SC}(\gamma) \quad = \quad \mathrm{L}\left[\frac{\gamma\left(\mu,\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(u)}\right]^{L-1} \quad \frac{1}{\Gamma(\mathrm{u})}\left[\left(t^{\mu-1}e^{-t}\right)\frac{\mu}{\bar{\gamma}^{\alpha/2}}\frac{d}{d\gamma}\gamma^{\alpha/2}\right] \tag{5.8}$$

$$f_{\gamma SC}(\gamma) \quad = \quad \mathrm{L}\left[\frac{\gamma\left(\mu,\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(u)}\right]^{L-1} \quad \frac{1}{\Gamma(\mathrm{u})}\left[\left(\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)^{\mu-1} e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{\alpha}{2}}} \frac{\mu}{\bar{\gamma}^{\alpha/2}}\frac{\alpha}{2}\gamma^{\frac{\alpha}{2}-1}\right] \tag{5.9}$$

$$f_{\gamma SC}(\gamma) = \mathrm{L}\left[\frac{\gamma\left(\mu,\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(u)}\right]^{L-1} \quad \frac{1}{2\Gamma(\mathrm{u})}\left[\mu^{\mu}\alpha\frac{\gamma^{\frac{\alpha\mu}{2}-1}}{\bar{\gamma}^{\frac{\alpha(\mu-1)}{2}}} e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{\alpha}{2}}} \frac{1}{\bar{\gamma}^{\alpha/2}}\right] \tag{5.10}$$

$$f_{\gamma SC}(\gamma) = \mathrm{L}\left[\frac{\gamma\left(\mu,\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(u)}\right]^{L-1} \quad \frac{1}{2\Gamma(\mathrm{u})}\left[\mu^{\mu}\alpha\frac{\gamma^{\frac{\alpha\mu}{2}-1}}{\bar{\gamma}^{(-\frac{\alpha\mu}{2})}} e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{\alpha}{2}}}\right] \tag{5.11}$$

Thus, the PDF of SNR of selection combining over α-μ faded channel with L- diversity path system is given as.

$$f_{\gamma SC}(\gamma) \quad = \quad \frac{\mathrm{L}\ \alpha\ \mu^{\mu}\ \gamma^{\frac{\alpha\mu}{2}-1}}{2\ \Gamma(u)\ \bar{\gamma}^{-\frac{\alpha\mu}{2}}}\ \mathrm{e}^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} \left[\frac{\gamma\left(\mu,\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(u)}\right]^{L-1} \tag{5.12}$$

Further solving eq.(5.12), using equation (8.352.6) of Gradshteyn and Ryzhik (2007), where the Incomplete Gamma function is represented by its equivalent finite series representation, as given below:

$$\gamma(n,x) \quad = \quad (n\text{-}1)! \left[1-e^{-x}\sum_{m=0}^{n-1}\frac{x^m}{m!}\right]$$

We get, $$f_{\gamma SC}(\gamma)=\frac{\mathrm{L}\ \alpha\ \mu^{\mu}\ \gamma^{\frac{\alpha\mu}{2}-1}}{2\ \Gamma(u)\ \bar{\gamma}^{\frac{\alpha\mu}{2}}}\ \mathrm{e}^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{\alpha}{2}}}\left(\frac{(\mu\text{-}1)!}{\Gamma(\mathrm{u})}\right)^{\mathrm{L}-1}\left[1-e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{\alpha}{2}}}\sum_{m=0}^{\mu-1}\frac{\left(\frac{\mu}{\bar{\gamma}^{\alpha/2}}\right)^m}{m!}\gamma^{\frac{\alpha m}{2}}\right]^{L-1} \quad (5.13)$$

$$f_{\gamma SC}(\gamma) \quad = \quad \frac{\alpha\ \mu^{\mu}\ (\mu\text{-}1)!\gamma^{\frac{\alpha\mu}{2}-1}}{\Gamma(u)^2\ \ \bar{\gamma}^{\frac{\alpha\mu}{2}}}\ \mathrm{e}^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{\alpha}{2}}}\left[1-e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{\alpha}{2}}}\sum_{m=0}^{\mu-1}\frac{\left(\frac{\mu}{\bar{\gamma}^{\alpha/2}}\right)^m}{m!}\gamma^{\frac{\alpha m}{2}}\right] \quad (5.14)$$

5.1.2 Switch & Stay Combining (SSC):

In selection combining, for systems which transmits continuously, will require a dedicated receiver for each branch to monitor the branch SNR continuously. SSC as stated by Simon and Alouini (2005), further simplifies the complexities of SC. In this in place of continually connecting the diversity path with best SNR, a particular diversity path is selected by the receiver till the SNR of that path drops below a predetermined threshold (γ_T). When a branch is chosen, the combiner outputs the signal as long as the SNR on that branch remains above the desired threshold (γ_T). When the SNR of the selected path drops below threshold (γ_T), then the receiver switches to another diversity path. This reduces the complexities relative to SC, because continuous and simultaneous monitoring of all diversity path in not required. In SSC also, co-phasing is not required, because only one branch output is taken at a time.

The CDF of SNR of SSC is given as

$$F_{\gamma_{SSC}} = \begin{cases} \Pr\left[\left(\gamma_1 \le \gamma_T\right)\text{and } \left(\gamma_2 \le \gamma\right)\right], & \gamma < \gamma_T \\ \Pr\left[\left(\gamma_T \le \gamma_1 \le \gamma\right) \text{ or } \left(\gamma_1 \le \gamma_T \text{ and } \gamma_2 \le \gamma\right)\right], & \gamma \ge \gamma_\mathrm{T} \end{cases} \quad (5.15)$$

Eq.(5.15) can be expressed in terms of CDF of individual branches

$$F_{\gamma_{SSC}} = \begin{cases} F_{\gamma}(\gamma_T) F_{\gamma}(\gamma), & \gamma < \gamma_T \\ F_{\gamma}(\gamma) - F_{\gamma}(\gamma_T) + F_{\gamma}(\gamma) F_{\gamma}(\gamma_T), & \gamma \geq \gamma_{\mathrm{T}} \end{cases} \tag{5.16}$$

PDF is obtained by differentiating CDF of eq.(5.16) as below

$$f_{\gamma_{SSC}} = \frac{\mathrm{d}\, \mathrm{F}_{\gamma\mathrm{SSC}}(\gamma)}{\mathrm{d}\gamma} = \begin{cases} F_{\gamma}(\gamma_T)\, f_{\gamma}(\gamma), & \gamma < \gamma_T \\ \left(1 + F_{\gamma}(\gamma_T)\right) f_{\gamma}(\gamma), & \gamma \geq \gamma_T \end{cases} \tag{5.17}$$

Since the SSC does not select the branch with the maximum SNR, the performance of SSC is between no diversity and selection combining.

5.1.3 Maximal Ratio Combining (MRC):

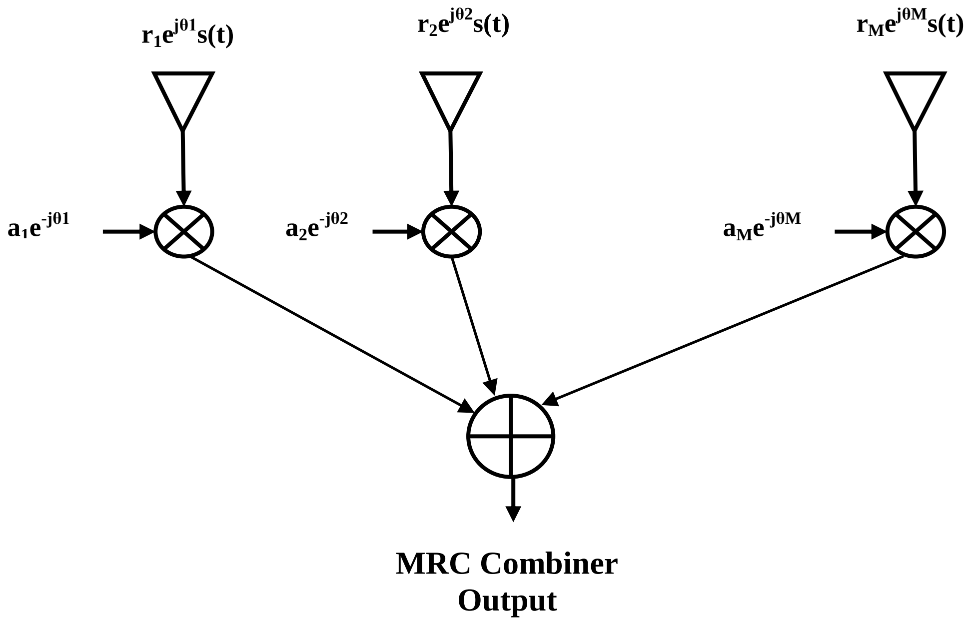

Fig.5.2. Co-phasing and summing for Maximal Ratio Combining

In SC and SSC, the combiner output is the signal on one of the branches, hence co-phasing was not required. In MRC, as stated by Goldsmith (2005), the output is weighted sum of all branches. Hence co-phasing as shown in fig 5.2 is required. Signals are co-phased by multiplying $a_i e^{-j\theta i}$, where θ_i is the phase of the incoming signal on the i^{th} branch. MRC is the most complex scheme in which all branches are optimally combined at the receiver. MRC requires scaling and co-phasing of individual branch as shown in fig.5.3. In this all the signals are weighted according to their individual signal to noise power ratios and then summed. Thus MRC produces an output SNR, which is equal to the sum of the individual SNRs. The advantage of MRC is producing an output with an acceptable SNR even when none of the individual signals are themselves acceptable. Best statistical reduction of fading is achieved by this technique. SNR of MRC for α-μ fading distribution is given as

$$\gamma_{MRC} = \left(R_1^{\,2} + R_2^{\,2}\right) \times SNR \tag{5.18}$$

Where R_1 and R_2 represent the fading envelope for two diversity paths seen by two different antennas. R_1^2 is fading power between transmitter and 1st receiving antenna. R_2^2 is fading power between transmitter and 2nd receiving antenna. SNR is unfaded signal to noise ratio.

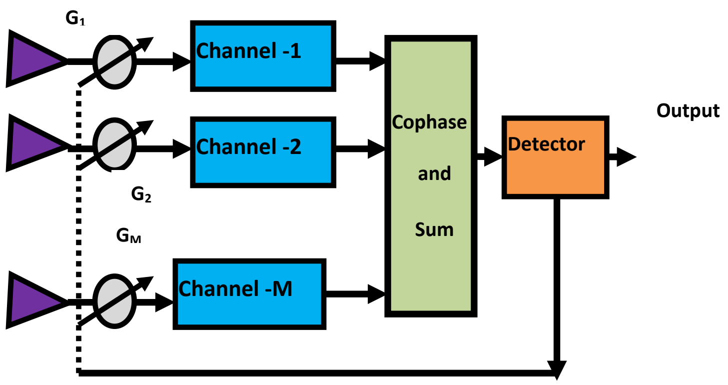

Fig.5.3. Maximal Ratio Combining

5.1.4 Equal Gain Combining (EGC):

A variant of MRC is equal gain combining (EGC), where signals from each branch are co-phased and then combined with equal magnitude weights. MRC requires knowledge of the time varying SNR on each branch, which may be difficult to measure. EGC method also has possibility like MRC of producing an acceptable output signal from a number of unacceptable input signals. Performance of EGC is quite close to MRC, having a very minor power penalty of less than 1dB. Thus, SNR improvement of EGC is better than selection combining but not better than MRC. SNR of EGC is given as

$$\gamma_{EGC} = \frac{1}{2}\left(R_1+R_2\right)^2 \times SNR \tag{5.19}$$

5.1.5 Performance Analysis of wireless link exploiting dual diversity with SC, SSC, EGC & MRC:

Outage performance and BER for different combining techniques for α-μ fading channel is obtained by Monte-Carlo simulation are shown in fig. 5.4. to fig.5.7. In these simulation 1000000 samples have been considered for a particular α and μ combination.

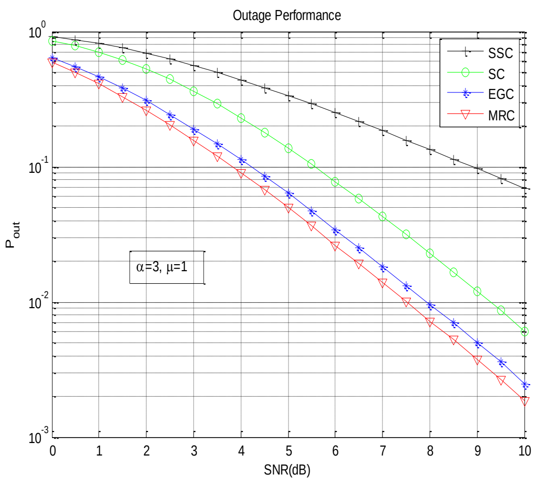

Fig. 5.4. Outage of α-μ fading for α=3 and μ=1

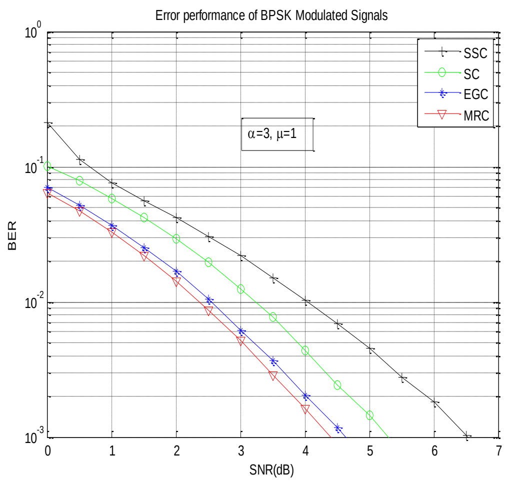

Fig. 5.5. BER of α-μ fading for α=3 and μ=1

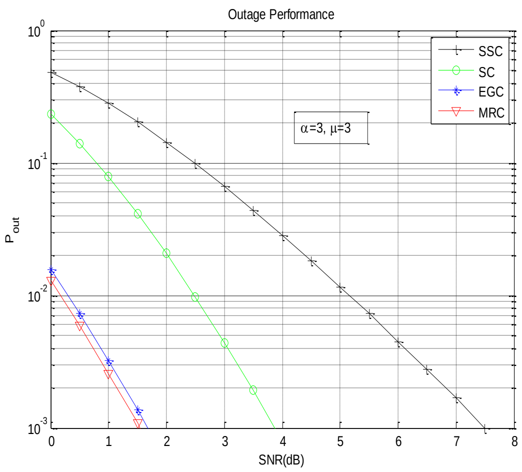

Fig. 5.6. Outage of α-μ fading for α=3 and μ=3

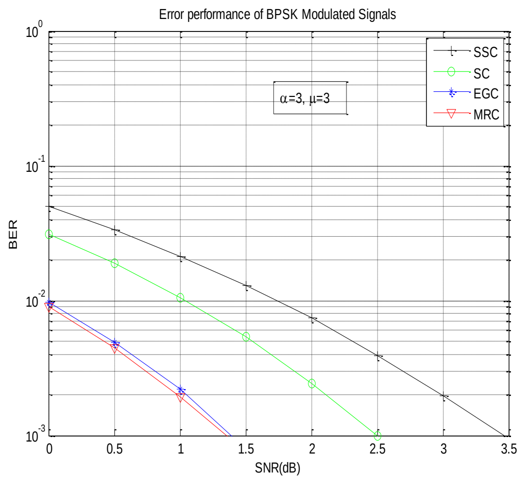

Fig. 5.7. BER of α-μ fading for α=3 and μ=3

In fig.5.4 and fig.5.5 outage performance and BER are shown for α=3, μ=1. It is observed that performance of MRC is better among all and subsequent performance is of EGC, SC and SSC.

In fig.5.6 and fig.5.7 outage and BER performance for α=3, μ=3 are shown. It is observed that performance of MRC is best among all and then the EGC, SC and SSC follow the performance order subsequently. The MRC is optimal combining scheme but it is at the expense of complexity. MRC requires knowledge of channel amplitude and phase, hence can be used for M-QAM or for any amplitude/phase modulations. On the other hand the SC combiner chooses the branch with highest SNR i.e. output is equal to the signal on only one of the branches, hence it does not require knowledge of the signal phases on each branch as in the case of MRC or EGC. The conventional SC is impractical because it requires the simultaneous and continuous monitoring of all the diversity branches. Therefore the SC is implemented in switched form i.e. SSC, where in place of continuously picking the best branch, receiver remains on a particular branch till its SNR drops below a specified threshold (γ_T=0.75). That is why SSC performance is slightly poor than SC.

Further, it is also seen that by increasing μ from 1 to 3, the improvement in outage and BER is observed. Increase in μ, indicates increase in clusters. Thus, inference can be drawn that by increasing the clusters outage and BER improves. In this analysis PDF, CDF, SNR, outage and BER of α-μ fading model have been briefly discussed. The simulated and analytical results of performance metrics such as outage and BER for MRC, EGC, SC and SSC spatial diversity combining schemes have been illustrated. The effect of α and μ parameters variation on BER and outage is brought out. Comparison between different diversity schemes have been discussed with help of simulation results. The result obtained have been published in a research paper ***"Performance Analysis of Space Diversity in α-μ Fading Channel***", in International Journal of Electrical and Electronics Engineers (IJEEE), ISSN 2321-2055, vol. 7, iss. 2, Jul - Dec 2015, pp.38-46.

5.2 Performance of Correlated Multi-Antenna *α-μ* Fading Channel:

5.2.1 Correlation Models in *α-μ* Fading Channel:

While studying the performance of diversity techniques, it is assumed that the signals considered for combining are independent of one another. The assumption of independence of channels facilitates the analysis but restricts the understanding of correlation phenomenon.

As per statistics theory variables are said to be correlated, when increase or decrease in one variable is accompanied by increase or decrease in other variable. Correlation is said to be positive when either both variables increase or both variables decrease. Correlation is said to be negative when one variable increases and other variable decreases. The degree of relationship between two correlated variables is measured as Correlation Coefficient (ρ).

$$\rho = \frac{\sum(x-X)(y-Y)}{\sqrt{\sum(x-X)^2 \times \sum(y-Y)^2}} \qquad 0 \le \rho < 1 \tag{5.20}$$

$$\rho = \frac{\frac{1}{n}\sum(x-X)(y-Y)}{\sqrt{\frac{\sum(x-X)^2}{n} \times \frac{\sum(y-Y)^2}{n}}} \tag{5.21}$$

$$\rho = \frac{cov(x,y)}{var(x)var(y)} = \frac{cov(x,y)}{\sigma_x \ \sigma_y} \tag{5.22}$$

In real-life scenario the assumption of channel independence is not valid due to various reasons such as insufficient antenna spacing in small-size mobile units. Further, for multipath diversity over frequency selective channels, correlation coefficients up to 0.6 between two adjacent paths in the channel impulse response of frequency selective channels have been observed by Simon and Alouini (2005). It has been established by statistical analysis of propagation of several macro-cellular, microcellular, and indoor wideband channel responses, that correlation coefficients is many times higher than 0.8, with no significant reduction in correlation even for large path delay differences. Hence, the maximum theoretical diversity gain promised by RAKE reception is not achieved. Therefore effect of correlation between combined signal must be taken into account, while carrying out analysis of performance of diversity techniques. Following α-μ Fading Correlation Models are discussed:

Model A: Two Correlated Branches with Non-identical Fading

Model A corresponds to the scenario of dual diversity reception over correlated α-μ fading channels that are not necessarily identically distributed i.e. mean and variance are different. This may therefore apply to small-size terminals equipped with space or polarization diversity where antenna spacing is insufficient to provide independent fading among signal paths.

$$\rho = \frac{\operatorname{cov}\left(r_1^2, r_2^2\right)}{\sqrt{\operatorname{var}\left(r_1^2\right)\operatorname{var}\left(r_2^2\right)}} \qquad 0 \le \rho < 1 \tag{5.23}$$

Where ρ is the power correlation coefficients between the two signals.

Model B: D Identically Distributed Branches with Constant Correlation

Model B corresponds to identically distributed α-μ fading channels. That means all channels are assumed to have the same average SNR per symbol and same fading parameter. This model assumes that the power correlation coefficient ρ is the same between all the channel pairs (d, d'= 1,2,…..,D) and is given as

$$\rho = \rho_{dd'} = \frac{\mathrm{cov}\left(r_d^2, r_{d'}^2\right)}{\sqrt{\mathrm{var}\left(r_d^2\right)\mathrm{var}\left(r_{d'}^2\right)}} \quad d \neq d' \quad 0 \leq \rho < 1 \tag{5.24}$$

This model may corresponds to the scenario of multichannel reception from closely spaced diversity antennas or three antennas placed on an equilateral triangle.

Model C: D Identically Distributed Branches with Exponential Correlation

Model C is proposed for identically distributed *α-μ* fading channels. That means all channels are assumed to have the same average SNR per symbol and same fading parameter. This model assumes that an exponential power correlation coefficient $\rho_{dd'}$ between any pair of channels (*d, d´*= 1,2,…..,D) is given by

$$\rho = \rho_{dd'} = \frac{\mathrm{cov}\left(r_d^2, r_{d'}^2\right)}{\sqrt{\mathrm{var}\left(r_d^2\right)\mathrm{var}\left(r_{d'}^2\right)}} = \rho^{|d-d'|} \qquad 0 \leq \rho < 1 \tag{5.25}$$

This model may corresponds to the scenario of multichannel reception from equi-spaced diversity antennas in which the correlation between the pairs of combined signals decays as the spacing between antennas increases.

Model D: D Non-identically Distributed Branches with Arbitrary Correlation

Model D is a very general model in which the combined branches may not be identically distributed *α-μ* fading channels and also may have an arbitrary correlation. This model assumes that the branches have an arbitrary average SNR per symbol and same fading parameter. The power correlation coefficient $\rho_{dd'}$ between any channel pair (*d, d´* = 1, 2, ….., D) is denoted by $\rho_{dd'}$.

The generality of this model may correspond to the impulse response of a frequency – selective channel with correlated paths and a non-uniform power delay profile. Thus, for wireless system performance and design study, characterization of the correlation properties of the signals in α-μ distribution is of utmost importance. Therefore determination of the joint probability density function as given by De Souza, Fraidenraich and Yacoub (2006) is significant.

Let R_1 and R_2 be two α-μ fading variates whose marginal statistics are respectively described as below: $\boldsymbol{R_1} \in \{\alpha_1, \mu_1, \hat{r}_1\}$ **and** $\boldsymbol{R_2} \in \{\alpha_2, \mu_2, \hat{r}_2\}$

and $0 \le \rho \le 1$ be a correlation parameter.

The joint PDF of the α-μ normalized envelopes, $P_1 = \frac{R_1}{\hat{r}_1}$ *and* $P_2 = \frac{R_2}{\hat{r}_2}$ is given as

$$f_{P_1,P_2}(p_1, p_2) = \frac{\alpha_1 \alpha_2 \mu^{\mu+1} p_1^{\frac{\alpha_1}{2}(\mu+1)-1} p_2^{\frac{\alpha_2}{2}(\mu+1)-1}}{(1-\rho)\rho^{\frac{\mu-1}{2}}\Gamma(\mu)} \times \exp\left(-\mu \frac{p_1^{\alpha_1} + p_2^{\alpha_2}}{1-\rho}\right) I_{\mu-1}\left(\frac{2\mu\sqrt{\rho p_1^{\alpha_1} p_2^{\alpha_2}}}{1-\rho}\right) \tag{5.26}$$

where $I_v(.)$ is the modified Bessel function of the first kind and order v.

Due to insufficient antenna spacing, desired signal envelopes R_1 and R_2 experience correlative α-μ fading with joint distribution:

$$f_{R_1,R_2}(R_1, R_2) = \frac{\alpha_1 \mu_1^{\mu_1} R_1^{\alpha_1\mu_1-1}}{\hat{R}_1^{\alpha_1\mu_1}\ \Gamma(\mu_1)}\ \exp\left[-\mu_1 \frac{R_1^{\alpha_1}}{\hat{R}_1^{\alpha_1}}\right] \times \frac{\alpha_2 \mu_1^{\mu_1} R_2^{\alpha_2\mu_1-1}}{\hat{R}_2^{\alpha_2\mu_1}\ \Gamma(\mu_1)}\ \exp\left[-\mu_1 \frac{R_2^{\alpha_2}}{\hat{R}_2^{\alpha_2}}\right]$$

$$\times \sum_{l=0}^{\infty} \frac{l!\Gamma(\mu_1)}{\Gamma(\mu_1+l)} \rho_{12d}{}^{l} L_l^{\mu_1-1} \times \left(\frac{\mu_1 R_1^{\alpha_1}}{\hat{R}_1^{\alpha_1}}\right) L_l^{\mu_1-1}\left(\frac{\mu_1 R_2^{\alpha_2}}{\hat{R}_2^{\alpha_2}}\right) \tag{5.27}$$

Let us consider a complex Gaussian random variable R_1 with variance σ^2. In order to generate a new complex Gaussian random variable R_2 with the same variance of R_1 and a correlation factor ρ between them, the equation for R_2 can be given as

$$R_2 = \rho R_1 + \left(1-\rho^2\right)^{\frac{1}{2}} v \tag{5.28}$$

where v is a complex Gaussian random variable with variance σ^2 .

Since correlation parameter ρ is $0 \le \rho \le 1$,

therefore when ρ is 0; then $R_2 = v$, i.e. R_2 is a complex Gaussian random variable but uncorrelated with R_1.

and when ρ is 1; then $R_2 = R_1$, i.e. R_2 is fully correlated with R_1.

Taking mean values of R_1 and v as zero and since both have the same variance σ^2, then the variance of R_2 is also σ^2 .

Generalized correlation coefficient ρ for two α-μ fading variates R_1 and R_2 is defined as

$$\rho = \frac{cov(R_1, R_2)}{\sqrt{\sigma_{R_1}{}^2 \times \sigma_{R_2}{}^2}} = \frac{cov(R_1, R_2)}{\sigma_{R_1} \times \sigma_{R_2}} \tag{5.29}$$

5.2.2 Performance of Correlated α-μ dual Diversity Combining:

Outage performance and BER for different combining techniques for correlated α-μ fading channel is obtained by Monte-Carlo simulation are shown in Fig. 5.8 to Fig.5.19. In these simulation 1000000 samples have been considered for a particular α and μ combination.

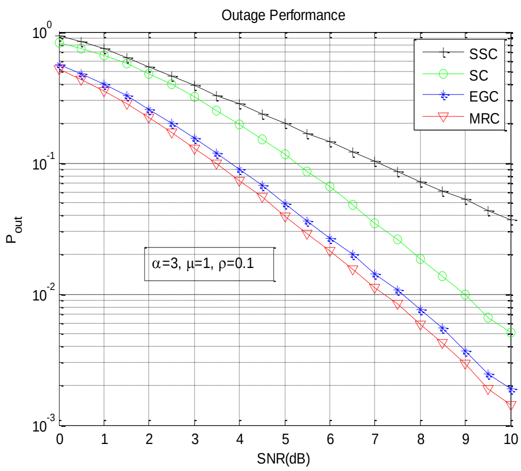

Fig. 5.8. Outage of α-μ fading for α=3, μ=1 & ρ= 0.1

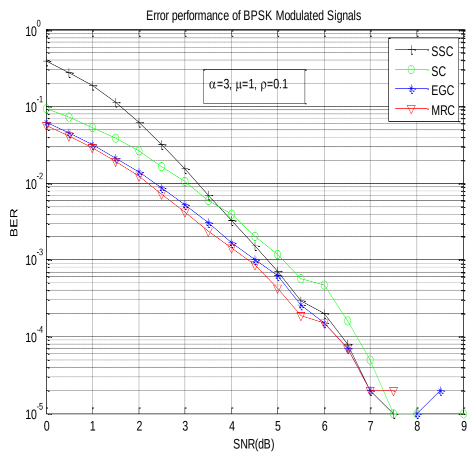

Fig. 5.9. BER of α-μ fading for α=3, μ=1 & ρ= 0.1

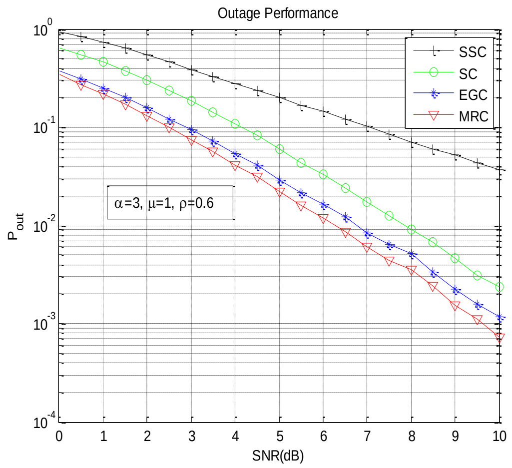

Fig. 5.10. Outage of α-μ fading for α=3, μ=1 & ρ= 0.6

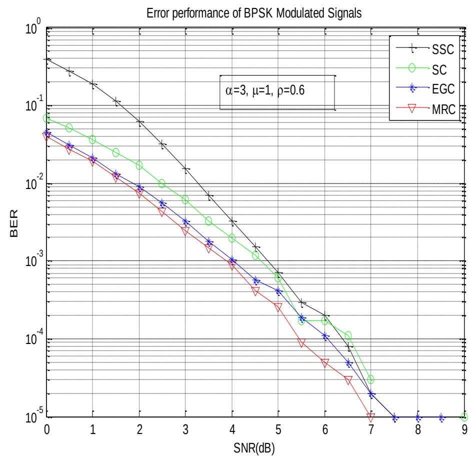

Fig. 5.11.BER of α-μ fading for α=3, μ=1 & ρ= 0.6

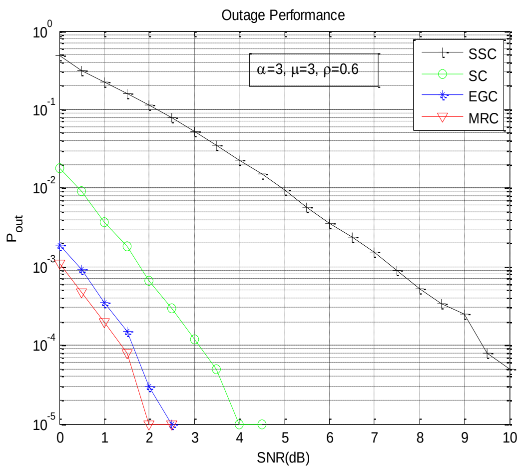

Fig. 5.12. Outage of α-μ fading for α=3, μ=3 & ρ= 0.6

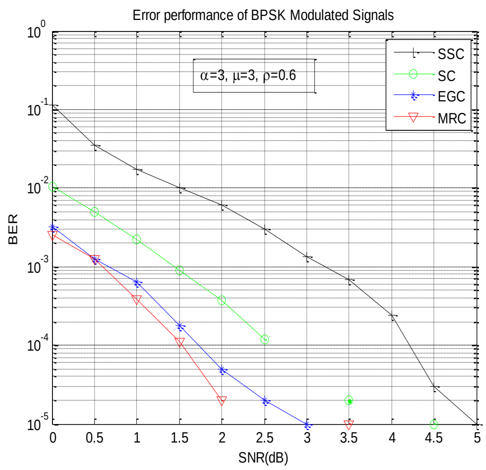

Fig. 5.13. BER of α-μ fading for α=3, μ=3 & ρ= 0.6

In fig.5.8 & fig.5.10 show the outage performance and fig. 5.9 & fig. 5.11 show the BER performance for α=3, μ=1 when correlation coefficient ρ increased from 0.1 to 0.6. It is seen that outage and BER performance for a given SNR has deteriorated with increase in ρ. In fig.5.10 & fig.5.12 show the outage performance and fig.5.11 & fig.5.13 show the BER performance for where α=3, ρ=0.6 when μ increased from 1 to 3, then it is seen that outage and BER performance improves drastically for a given SNR. It is noticed that by increasing μ from 1 to 3, the improvement in outage and BER is observed. Increase in μ, indicates increase in clusters. Thus, inference can be drawn that by increasing the clusters outage and BER improves for the same correlation coefficient.

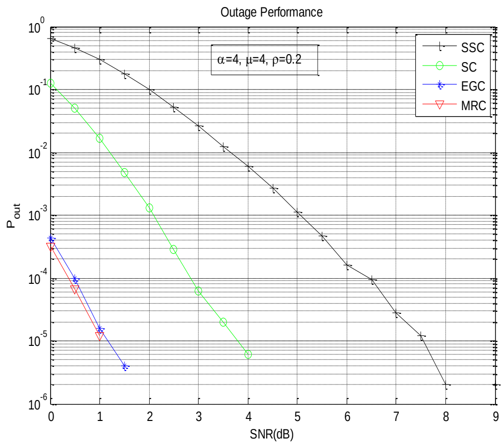

Fig. 5.14. Outage of α-μ fading for α=4, μ=4 & ρ= 0.2

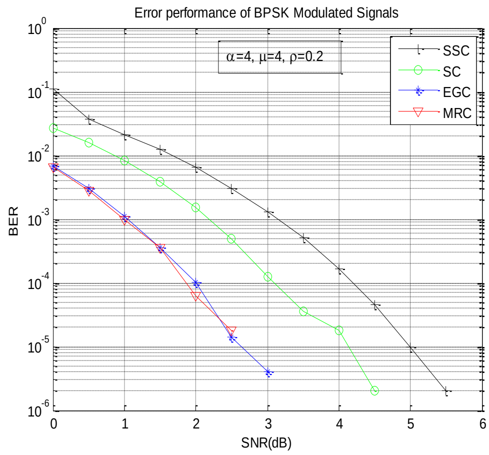

Fig. 5.15. BER of α-μ fading for α=4, μ=4 & ρ= 0.2

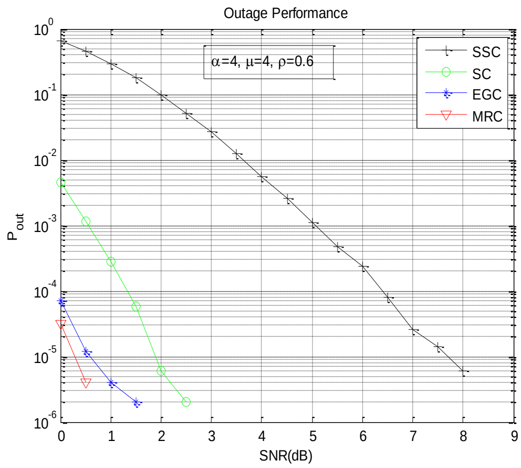

Fig. 5.16. Outage of α-μ fading for α=4, μ=4 & ρ= 0.6

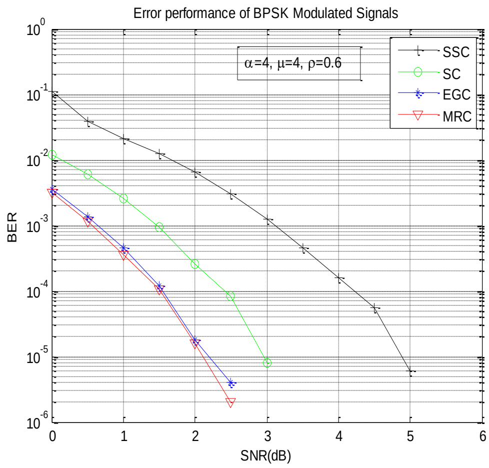

Fig. 5.17. BER of α-μ fading for α=4, μ=4 & ρ= 0.6

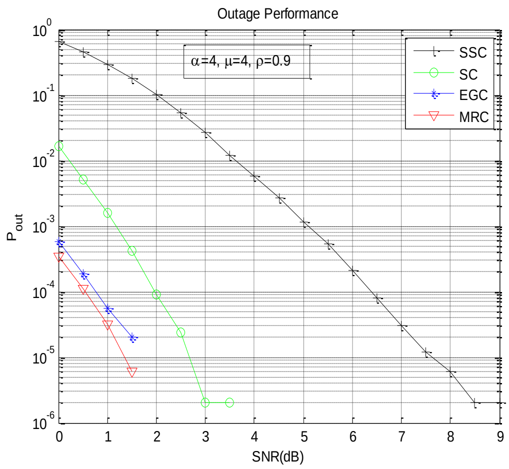

Fig. 5.18. Outage of α-μ fading for α=4, μ=4 & ρ= 0.9

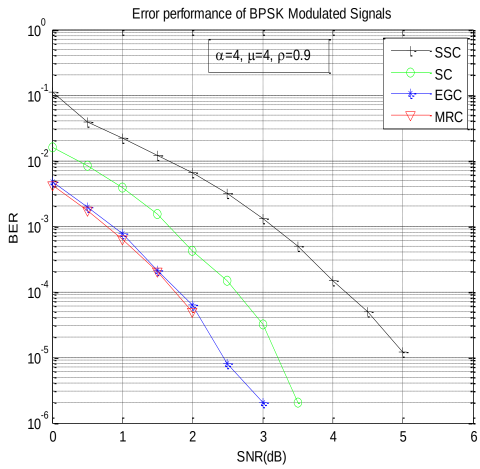

Fig. 5.19. BER of α-μ fading for α=4, μ=4 & ρ= 0.9

In fig.5.14, fig.5.16 & fig.5.18 show the outage performance and fig.5.15, fig.5.17 & fig.5.19 show the BER performance for α=4 and μ=4 when ρ increases from 0.2 to 0.6 to

0.9. It is seen that outage and BER performance deteriorates for a given SNR when ρ increases. Thus it may be concluded that increase in ρ has adverse effect on outage and BER for a given α and μ parameters. It is also observed that in dual diversity correlated fading performance of MRC is better among all and then the EGC, SC and SSC follow the performance subsequently. The MRC is optimal combining scheme but it is at the expense of complexity. MRC requires knowledge of channel amplitude and phase, hence can be used for M-QAM or for any amplitude/phase modulations. On the other hand the SC combiner chooses the branch with highest SNR i.e. output is equal to the signal on only one of the branches, hence it does not require knowledge of the signal phases on each branch as in the case of MRC or EGC. The conventional SC is impractical because it requires the simultaneous and continuous monitoring of all the diversity branches. Therefore the SC is implemented in switched form i.e. SSC, where in place of continuously picking the best branch, receiver remains on a particular branch till its SNR drops below a specified threshold (γ_T=0.75). That is why SSC performance is slightly poor than SC.

In this analysis α-μ fading model and probability density function have been briefly discussed. The simulated and analytical results of performance metrics such as outage and BER for α-μ fading correlation schemes have been illustrated. The effect of α and μ parameters and correlation coefficient variation on BER and outage is brought out. The result obtained have been published in a research paper ***"Performance Analysis of Dual Diversity Receiver over Correlated α-μ Fading Channel***", International Journal of Advance Research in Science and Engineering (IJARSE), ISSN 2319-8354, vol. 4, iss. 8, Aug 2015, pp.92-100.

5.3 Performance of Correlated Multi-Antenna α-μ Channel:

Outage performance and BER for different combining techniques for correlated α-μ fading channel for multi-antenna system obtained by Monte-Carlo simulation are shown in fig. 5.20 to fig.5.31. In these simulation 1000000 samples have been considered for each curve shown. As observed in fig. 5.20 to fig. 5.22, α =7/4, μ=2, ρ=0.99, and when number of receiving antennas R_x is varied from 3 to 5, the outage probability decreases i.e. performance of system improves for desired average SNR.

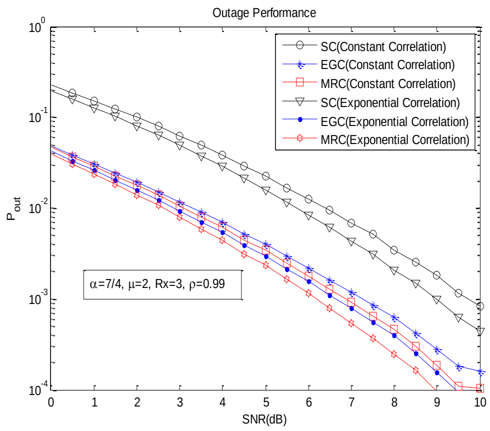

Fig. 5.20. Outage for α=7/4, μ=2, R_x =3 & ρ= 0.99 for different combining.

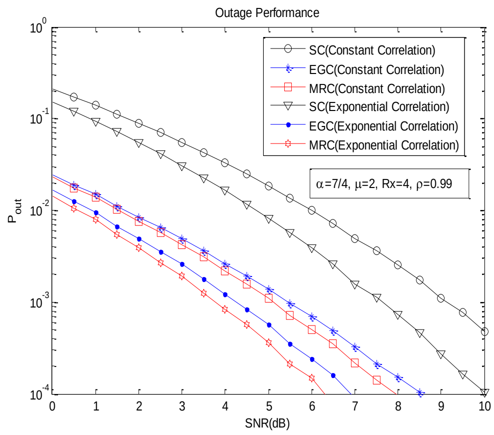

Fig. 5.21. Outage for α=7/4, μ=2, R_x =4 & ρ= 0.99 for different combining.

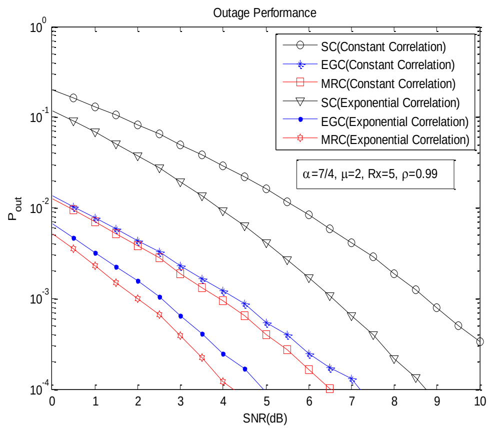

Fig. 5.22. Outage for α=7/4, μ=2, R_x =5 and ρ= 0.99 for different combining.

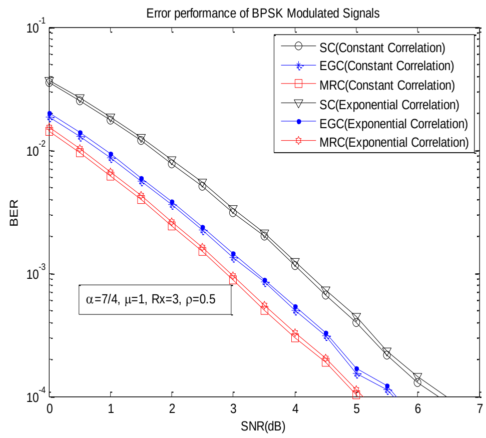

Fig. 5.23. BER for α=7/4, μ=1, R_x =3 and ρ= 0.5 for different combining.

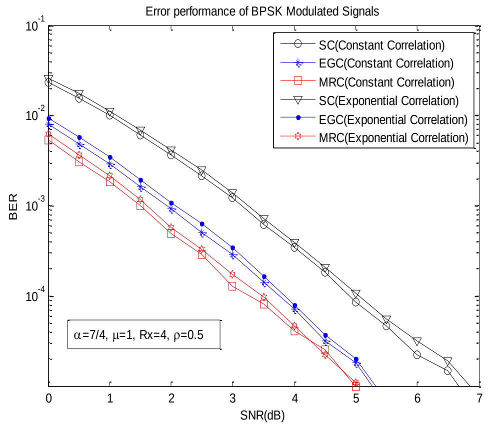

Fig. 5.24. BER for α=7/4, μ=1, R_x=4 and ρ= 0.5 for different combining.

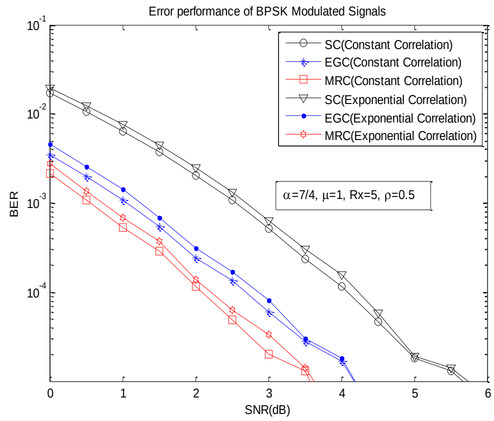

Fig. 5.25. BER for α=7/4, μ=1, R_x=5 and ρ= 0.5 for different combining.

It is seen in fig. 5.23 to fig. 5.25, α =7/4, μ=1, ρ=0.5, and by varying number of receiving antennas R_x from 3 to 5, the bit error rate (BER) decreases i.e. performance of system improves for desired average SNR.

In fig.5.26 to fig.5.28, where α =7/4, μ=1, and ρ=0.99, when number of receiving antennas R_x is varied from 3 to 5, the outage probability again decreases as desired for a given average SNR. It is worth noting that in this case (μ=1) outage is higher than the corresponding values of previous (μ=2) case. In fig. 5.22 (μ=2) for 1dB average SNR, for MRC exponential correlation outage is 2.5×10^{-3}, whereas from fig. 5.28 (μ=1) for 1dB average SNR, the respective outage is 1.5×10^{-1}, which is a poor performance comparatively. Thus multipath cluster parameter μ, plays an important role in outage performance of wireless network.

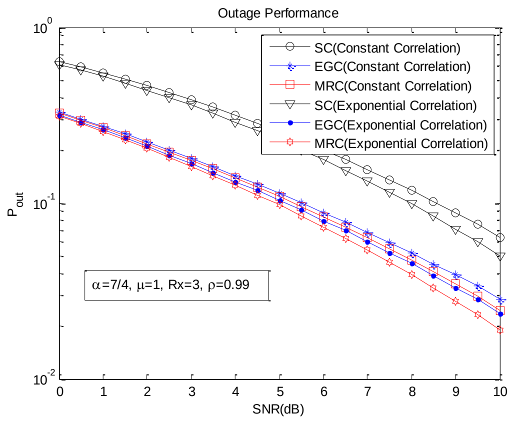

Fig. 5.26. Outage for α=7/4, μ=1, R_x =3 and ρ= 0.99 for different combining.

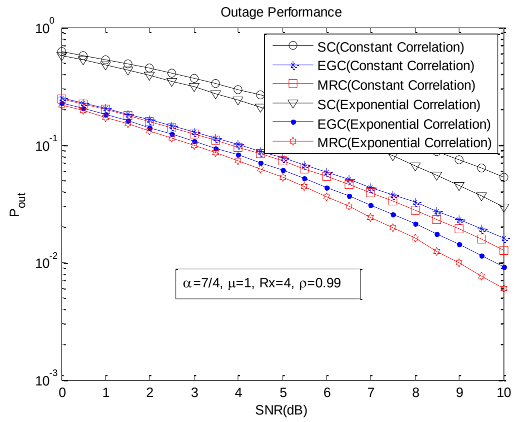

Fig. 5.27. Outage for α=7/4, μ=1, R_x =4 and ρ= 0.99 for different combining.

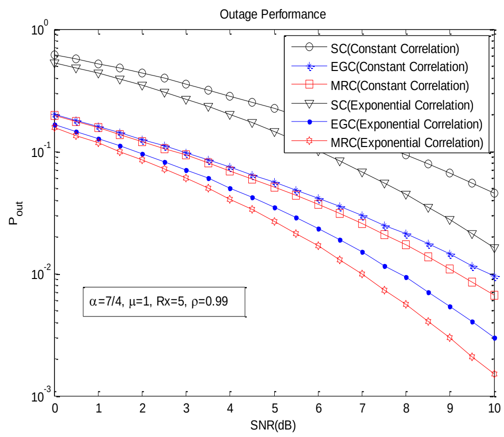

Fig. 5.28. Outage for α=7/4, μ=1, R_x =5 and ρ= 0.99 for different combining.

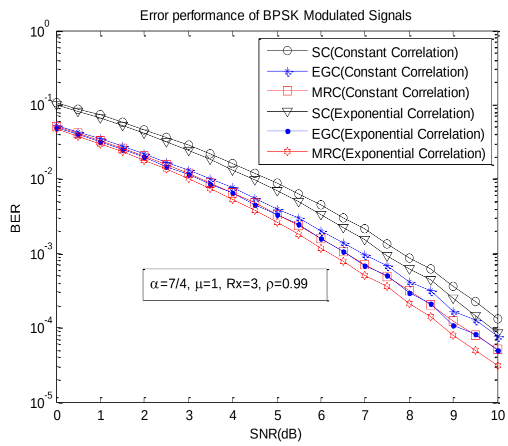

Fig. 5.29. BER for α=7/4, μ=1, R_x =3 and ρ= 0.99 for different combining.

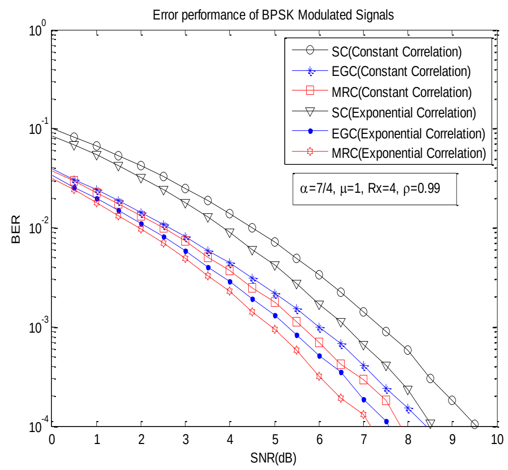

Fig. 5.30. BER for α=7/4, μ=1, R_x =4 and ρ= 0.99 for different combining.

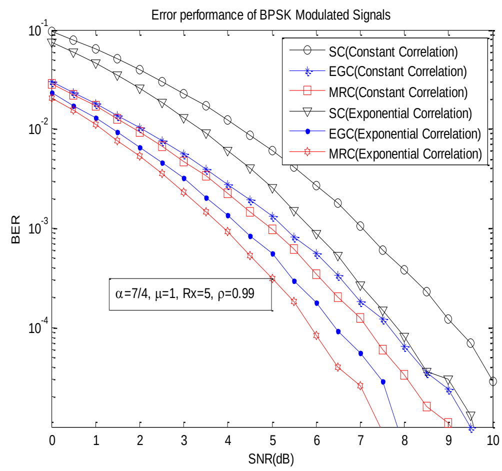

Fig. 5.31. BER for α=7/4, μ=1, R_x =5 and ρ= 0.99 for different combining.

In a similar way regarding BER for α =7/4, μ=1, and ρ=0.99 shown in fig. 5.29 to fig. 5.31, by varying number of receiving antennas R_x from 3 to 5, the bit error rate (BER) decreases i.e. performance of system improves for desired average SNR. But again it can be seen that BER decrease is not as it was for μ=2 case. Regarding correlation coefficient, we notice from fig.5.25 (α =7/4, μ=1, R_x=5, ρ=0.5) at 2dB average SNR, BER is 1.5×10^{-4} for MRC exponential and from fig.5.31 (α =7/4, μ=1, R_x=5, ρ=0.99) at 2dB average SNR, corresponding BER is 6×10^{-3}. Thus BER is higher for high value of correlation. It is also observed that in multiple antenna diversity correlated fading performance of MRC is better among all and then the EGC and SC follows the performance subsequently. Also, performance of exponential correlation is better than constant correlation. The MRC is optimal combining scheme but it is at the expense of complexity. On the other hand the SC combiner chooses the branch with highest SNR i.e. output is equal to the signal on only one of the branches, hence it does not require knowledge of the signal phases on each branch as in the case of MRC or EGC.

In this analysis α-μ fading model and probability density function have been briefly discussed. The simulated and analytical results of performance metrics such as outage and BER for α-μ fading correlation multi-antenna system have been illustrated. The effect of number of antenna variation and constant & exponential correlation coefficient variation on BER and outage is brought out. The result obtained in this analysis have been documented in a research paper titled, ***"Performance Analysis of Multi Antenna Receiver over Correlated α-μ Fading Channel***", which is published in the International Journal of Innovative Research in Computer and Communication Engineering (IJIRCCE), ISSN 2320-9801, vol.3, iss.9, Sep. 2015, pp.8429-8436. (*DOI: 10.15680/IJIRCCE.2015. 0309105*).

CHAPTER 6

COOPERATIVE COMMUNICATION

The multiple-input-multiple-output (MIMO) systems using multiple antennas at the transmitter as well as the receiver have shown tremendous improvements in communication network reliability. However due to size, cost and hardware constraints the use of MIMO techniques may not always be feasible especially in small devices. It is because, antenna spacing of half the carrier wavelength is required to ensure uncorrelated signals. Therefore, although multi-antenna systems improves diversity gain of wireless network, but is not suitable for small wireless nodes due to limited hardware and signal processing capability. This has led to multiple relay communication system which involves making multi-relay nodes cooperatively transmit and receive by forming virtual antenna arrays. This method is broadly named as cooperative communication. The need of high-data-rate, spectrally efficient and reliable wireless communication can be met by relay-based cooperative communication. The diversity can be achieved through user cooperation, where mobile users share their physical resources to create a virtual array of relays, which eliminates the necessity of multiple-antenna on wireless terminal.

Relay Transmission topology can be serial relay transmission or parallel relay transmission. In serial relay signal propagates from one relay to another relay hence may face multi-path fading. Parallel relay transmission overcomes the problem by propagating the signal through multiple relay path in same hop and destination combines the signal by the combining technique. The adaptive cooperation relay systems improves performance by eliminating noise/error propagation. This system adapts their transmission format according to channel condition between source and relay. The relay nodes process the signal received from the source if received SNR at relay is more than a threshold, else the relay remains the silent. When relay remains silent, the source may retransmit the signal to destination or can choose more powerful code to mitigate fading.

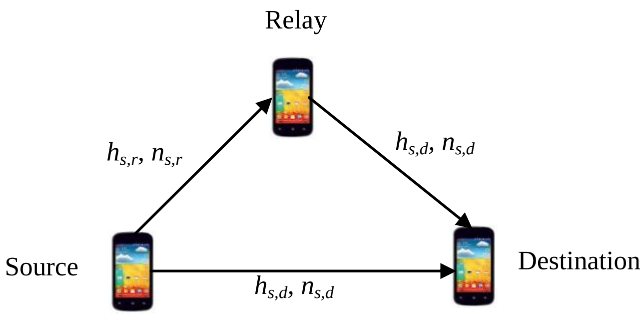

Fig.6.1. Two-hop Relay Cooperative Communication

In fig.6.1 two-hop relay cooperative communication system is shown, here two copies of signal is received at destination one through direct link (DL) from source and other through relay link from source-relay-destination.

6.1 End-to-End Performance of Relay-based Communication System:

In a two-hop wireless communication system with relay shown in fig.6.2, the signal is transmitted as broadcast through relay in between source and destination

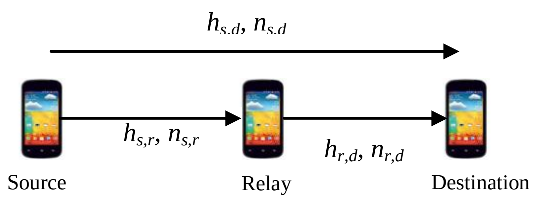

Fig.6.2. Two-hop Relay-based Wireless Communication system

6.1.1 Amplify and Forward:

If in the communication system shown in fig.6.2, the relay receives the signal in first hop, amplifies it and then transmits in next hop, it is called amplify and forward (AF) mode of relaying. The destination will receive two version of the same information one through the relay and other direct transmission between the source and destination. AF requires very less processing by the relays which makes them fast in terms of delay, hence suitable for delay sensitive signal such as voice or live video. Noise amplification also occurs along with signal, which is a serious issue in AF operation. Assuming that source transmitting signal x, which is received by relay ($y_{s,r}$) and destination ($y_{s,d}$) respectively as

$$y_{s,r} = h_{s,r}x + n_{s,r} \tag{6.1}$$

$$y_{s,d} = h_{s,d}x + n_{s,d} \tag{6.2}$$

where

$h_{s,r}$, $h_{r,d}$ & $h_{s,d}$ = fading amplitude of the $\alpha-\mu$ wireless channels; source-to-relay, relay-to-destination & source-to-destination respectively.

$n_{s,r}$, $n_{r,d}$ & $n_{s,d}$ = Additive White Gaussian Noise (AWGN) of the $\alpha-\mu$ wireless channels; source-to-relay, relay-to-destination & source-to-destination, respectively with variance N_0.

The relay also equalizes the influence of the fading channel between the source and the relay. This will be accomplished by scaling the received signal by a factor G. Thus, the signal received by relay is forwarded to destination as ($y_{r,d}$).

$$y_{r,d} = Gh_{r,d}y_{s,r} + n_{r,d} \tag{6.3}$$

The received signal at destination through relay link may be simplified from eq. (6.3) as

$$\begin{aligned} y_{r,d} &= h_{r,d}G\ y_{s,r} + n_{r,d} \\ &= h_{r,d}G\left\{h_{s,r}x + n_{s,r}\right\} + n_{r,d} \\ &= h_{s,r}h_{r,d}Gx + h_{r,d}Gn_{s,r} + n_{r,d} \end{aligned} \tag{6.4}$$

Thus, SNR at destination for relay link from eq. (6.4) will be

$$\text{SNR}=\frac{\text{Signal power}}{\text{Noise power}} = \frac{(h_{s,r}h_{r,d}G)^2}{(h_{r,d}G)^2 N_0 + N_0} \tag{6.5}$$

$$\text{SNR} = \frac{h_{s,r}^2 h_{r,d}^2}{h_{r,d}^2 N_0 + N_0/G^2} = \frac{\dfrac{h_{s,r}^2}{N_0}\dfrac{h_{r,d}^2}{N_0}}{\dfrac{h_{r,d}^2}{N_0} + \dfrac{1}{G^2 N_0}} \tag{6.6}$$

$$\text{Let, SNR}= \gamma_i = \frac{h_i^2}{N_0}, \text{ and } G^2 = \frac{1}{h_{s,r}^2 + N_0} \tag{6.7}$$

Therefore SNR for AF relay link from eq. (6.6) & (6.7) will be,

$$\gamma_{AF} = \frac{\gamma_{s,r}\gamma_{r,d}}{\gamma_{s,r} + \gamma_{r,d} + 1} \tag{6.8}$$

Similarly, SNR for direct link from eq. (6.2),

$$\gamma_{\mathrm{DL}} = \gamma_{s,d} = \frac{h_{s,d}^2}{N_0} \tag{6.9}$$

Since $(\gamma_{s,r} + \gamma_{r,d}) > 1,$ hence SNR for AF will be

$$\gamma_{AF} = \frac{\gamma_{s,r}\gamma_{r,d}}{\gamma_{s,r} + \gamma_{r,d}} = \frac{1}{\frac{1}{\gamma_{s,r}} + \frac{1}{\gamma_{r,d}}} \tag{6.10}$$

Outage and BER for AF may be given as

$$P_{out}\big|_{AF} = \int_0^{\gamma_{th}} f_{\gamma_{AF}}(\gamma)\, d\gamma \tag{6.11}$$

$$BER\left(P_e\big|_{AF}\right) = \int_0^{\infty} Q\left(a\sqrt{\gamma}\right) f_{\gamma_{AF}}(\gamma)\ d\gamma \tag{6.12}$$

where γ_{th} is threshold SNR, a is constant,
γ_{AF} are end to end received SNR for AF.

It is seen that in AF mode with two-hop system, the two copies of signal *x* is received by the destination one directly from the source and other through relay links. These two signals can be combined in many different combining techniques. One of the optimum technique that maximize the overall signal to noise ratio is called the maximal ratio combiner (MRC). The maximum feasible end-to-end transmission rate for two-hop AF relaying scheme with MRC diversity combining may be given as

$$\begin{aligned} C &= \frac{1}{2}\log_2\left(1 + SNR\right) = \frac{1}{2}\log_2\left(1 + \gamma_{s,d} + \gamma_{s,r,d}\right) \\ &= \frac{1}{2}\log_2\left(1 + \gamma_{s,d} + \frac{\gamma_{s,r}\gamma_{r,d}}{\gamma_{s,r} + \gamma_{r,d} + 1}\right) \end{aligned} \tag{6.13}$$

Thus, end-to-end instantaneous signal-to-noise ratio (SNR) γ_{end} of AF multi-hop transmission over generalized fading channels may be characterized from eq. (6.10) simply as the normalized harmonic mean of the transmission hops' instantaneous SNRs, and is given by

$$\gamma_{end} = \frac{1}{\frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \ .\ .\ .\ . + \frac{1}{\gamma_L}} \tag{6.14}$$

where L is the number of hops in the multi-hop transmission and γ_i denotes the instantaneous SNR of the i^{th} hop.

The major advantages of the AF method are the good diversity gain, better performance than direct transmission and decode-and-forward and also high capacity with increase in number of relays can be achieved.

6.1.2 Decode and Forward:

The decode and forward (DF) is a digital and regenerative scheme, where relay receives the signal, decodes it and after encoding retransmit it to the destination. Noise does not propagate, because the noise will not be amplified and it is excluded by the decoding process. However, when a decoding error occurred at the relay node due to the deep fading in channel between the source and the relay, this can be considered as the major problem with decode and forward method. The problem will be worsen if detection at the relay node is also unsuccessful and will result in bad performance. Due to the broadcast nature of the wireless medium, the relay and the destination nodes will receive noisy copy of the signals. In DF method processing time is higher causing delay, hence DF is not suitable for delay sensitive signals. Bhatnagar and Hjorungnes, (2011) have presented most likelihood decoder for DF based cooperative communication system.

The signal received at relay from source is given by eq. (6.1). The signal directly received at destination from source is given by eq. (6.2). The signal is decoded at relay and encoded. When signal x is estimated by relay, the encoded signal obtained by maximum likelihood detector will be

$$\hat{x} = \arg\min_{x} \left| y_{s,r} - h_{s,r} x \right|^2 \tag{6.15}$$

Hence, the signal received in DF, at destination will be

$$y_{r,d} = h_{r,d}\hat{x} + n_{r,d} \tag{6.16}$$

The SNR at destination for DF relay link will be

$$\gamma_{DF} = \min\left(\gamma_{s,r},\ \gamma_{r,d}\right) \tag{6.17}$$

The CDF of SNR for source-relay-destination DF link, is given as

$$F_{\gamma_{DF}}(\gamma) = 1 - \left[\left\{1 - F_{\gamma_{sr}}(\gamma)\right\}\left\{1 - F_{\gamma_{rd}}(\gamma)\right\}\right] \tag{6.18}$$

Assuming both links source-relay and relay-destination identical, for simplicity in analysis, hence CDF for both link equal to $F_\gamma(\gamma)$, therefore we get

$$F_{\gamma_{DF}}(\gamma) = 1 - \left[\left\{1 - F_\gamma(\gamma)\right\}^2\right] \tag{6.19}$$

With help of PDF of SNR of a-μ fading given in eq. (3.56), the CDF is obtained

$$F_\gamma(\gamma_{th}) = \int_0^{\gamma_{th}} f_\gamma(\gamma)\,d\gamma = \int_0^{\gamma_{th}} \frac{\alpha\ \mu^\mu\ \gamma^{\frac{\alpha\mu}{2}-1}}{2\,\Gamma(\mu)\,\overline{\gamma}^{\alpha\mu/2}}\ e^{-\mu\left(\frac{\gamma}{\overline{\gamma}}\right)^{\alpha/2}}\,d\gamma \tag{6.20}$$

On solving eq.(6.20), we get CDF eq.(3.62) for identical links as

$$F_\gamma(\gamma_{th}) = \frac{\gamma\left(\mu,\ \mu\left(\frac{\gamma_{th}}{\overline{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(u)} \tag{6.21}$$

Now, CDF of SNR for source-relay-destination DF link, from eq.(6.19) will be

$$F_{\gamma_{DF}}(\gamma_{th}) = 1 - \left[\left\{1 - \frac{\gamma\left(\mu,\ \mu\left(\frac{\gamma_{th}}{\overline{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(u)}\right\}^2\right] \tag{6.22}$$

The CDF obtained in eq.(6.22) is the same as outage probability for source-relay-destination DF link. Therefore

$$P_{out}\Big|_{DF} = 1 - \left[\left\{ 1 - \frac{\gamma\left(\mu,\ \mu \left(\frac{\gamma_{th}}{\bar{\gamma}} \right)^{\alpha/2} \right)}{\Gamma(u)} \right\} \right]^2 \tag{6.23}$$

CDF of eq.(6.22), when differentiated will result in PDF of source-relay-destination DF link. Therefore, the BER will be given as

$$P_e\Big|_{DF} = \int_0^\infty Q\left(a\sqrt{\gamma}\right) f_{\gamma_{DF}}(\gamma)\ d\gamma \tag{6.24}$$

where a is a constant and depends on modulation and detection combination.

The transmission rate in DF case will be bounded by the capacity of the two links i.e source – destination direct link, and source – relay – destination link. Being a delay sensitive channel due to decoding/encoding at relay node, the maximum end-to-end achievable rate in DF will be

$$C = \frac{1}{2}\min\left\{\log_2(1+\gamma_{s,d}), \log_2(1+\gamma_{s,r}+\gamma_{r,d})\right\} \tag{6.25}$$

There is also decode and re-encode (DR) scheme where code used at relay for encoding the message is different than that used at source. Thus in this DR scheme, destination receives two copies of the same message encoded with different codes.

6.1.3 Path Loss in Radio Propagation:

The free space radio propagation model presented by Rappaport (2011) is helpful in predicting the received signal strength, when there is clear line-of-sight path between transmitter and receiver. The power received by the receiver antenna $P_r(d)$ is given by Friis free space model as

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L} \tag{6.26}$$

Where, $P_r(d)$ is received power which is a function of transmitter-receiver separation d in meters, P_t is transmitted power, G_t is transmitter antenna gain, G_r is receiver antenna gain, λ is wavelength in meters, L is the system loss factor.

Path loss represents signal attenuation as a positive quantity measured in dB. Path loss is defined as the difference (in dB) between the effective transmitted power and received power. It may or may not include the effect of the antenna gains. The path loss when antenna gains are included is given from eq.(6.26) as

$$PL(\mathrm{dB}) = 10\log\frac{P_t}{P_r} = 10\log\left[\frac{\left(4\pi^2\right)d^2}{G_t G_r \lambda^2}\right] = -10\log\left[\frac{G_t G_r \lambda^2}{\left(4\pi^2\right)d^2}\right] \tag{6.27}$$

Here L =1, considering that there is no loss in system hardware.

If antenna gains are excluded i.e. assuming the antenna gains as unity, then the path loss will be

$$PL(\mathrm{dB}) = -10\log\left[\frac{\lambda^2}{\left(4\pi^2\right)d^2}\right] \tag{6.28}$$

It has been established by theoretical and measurement-based propagation models that average received signal power decreases logarithmically with distance, irrespective of outdoor or indoor radio channels. The average large scale path loss for a given transmitter-receiver separation (d) is expressed by using a path loss exponent (n).

$$\overline{PL}(d) \;\; \alpha \;\; \left(\frac{d}{d_0}\right)^n \tag{6.29}$$

or

$$\overline{PL}(d) \; = \; \overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right) \tag{6.30}$$

Where d_0 is the close-in reference distance, which is the possible smallest distance from the transmitter, considered for practical usage in mobile communication. The value of path loss exponent (n) depends on the specific propagation environment. For example in free space n is equal to 2, and in urban obstructions n may be up to 6.

6.1.4 Performance Analysis in AF and DF Mode over α-μ Channel:

Outage and BER performance for two-hop relay based system over α-μ fading channel is obtained by Monte-Carlo simulation. The communication system shown in fig.6.2 has been considered in this simulation. Outage probability and BER performance simulated results for different α and μ values, and path loss (PL) condition are shown in fig. 6.3 to fig.6.10. In these simulation 100000 bits have been considered for a particular α and μ combination. In direct link between source-destination BPSK modulation is used, whereas in relay link QPSK modulation is used to maintain the same data rate for honest performance comparison. In fig.6.3 the outage performance for path loss exponent n=3, α=3, μ=2 is shown. It is seen that at 10 dB SNR, the outage for direct link (DL) is highest as 5.0×10^{-2}, and that for amplify and forward (AF) is 1.3×10^{-2}, and for decode and forward (DF) is 6.0×10^{-3}. In fig.6.5 the PL exponent has increased to 4, than fig.6.3, other parameters are same, therefore as a result outage of DL has deteriorated to 2.7×10^{-1}, but there is no change in outage of AF and DF.

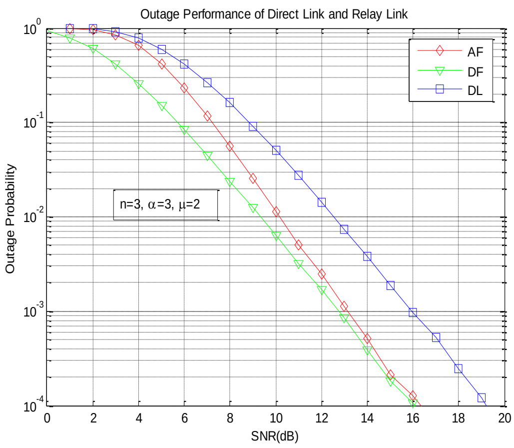

Fig. 6.3. Outage of two-hop relay system over α-μ fading for path loss exponent n=3, α=3, μ=2

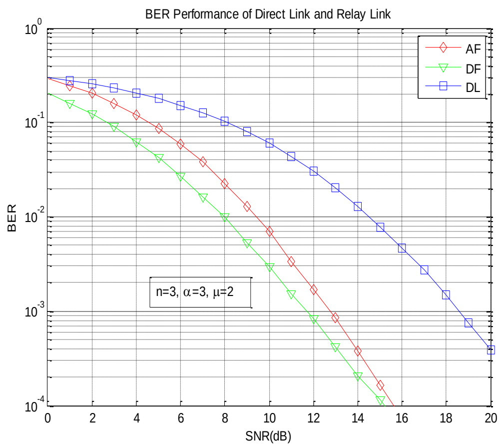

Fig. 6.4. BER of two-hop relay system over α-μ fading for path loss exponent n=3, α=3, μ=2

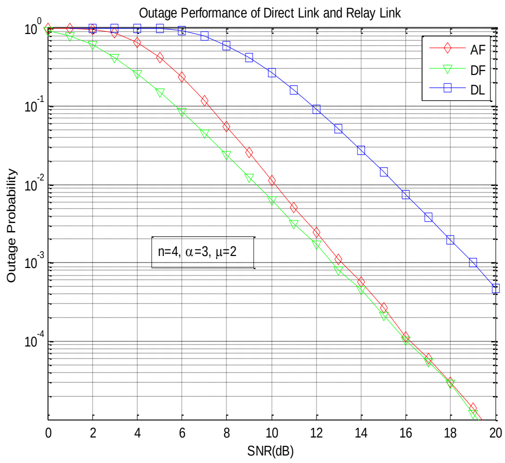

Fig. 6.5. Outage of two-hop relay system over α-μ fading for path loss exponent n=4, α=3, μ=2

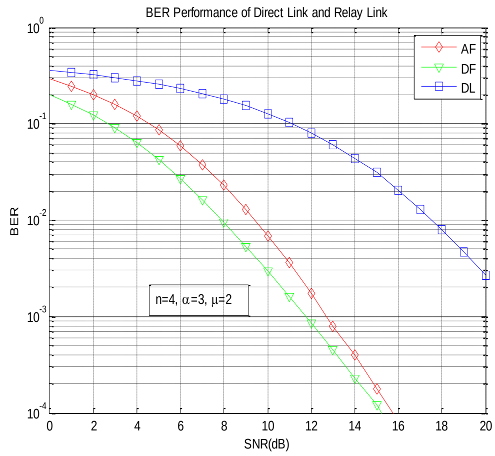

Fig. 6.6. BER of two-hop relay system over α-μ fading for path loss exponent n=4, α=3, μ=2

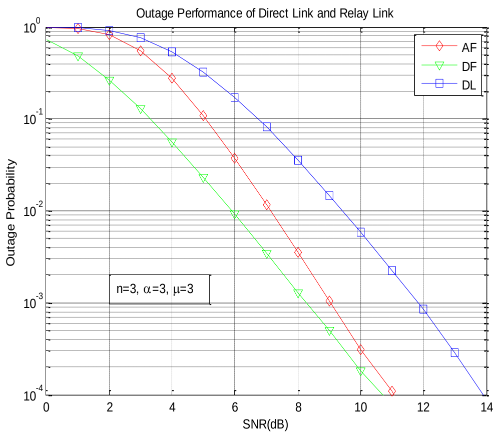

Fig. 6.7. Outage of two-hop relay system over α-μ fading for path loss exponent n=3, α=3, μ=3

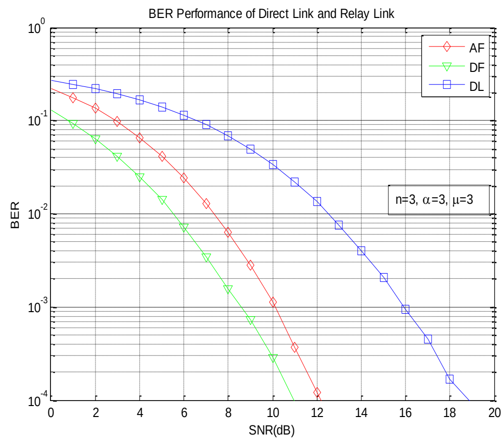

Fig. 6.8. BER of two-hop relay system over α-μ fading for path loss exponent n=3, α=3, μ=3

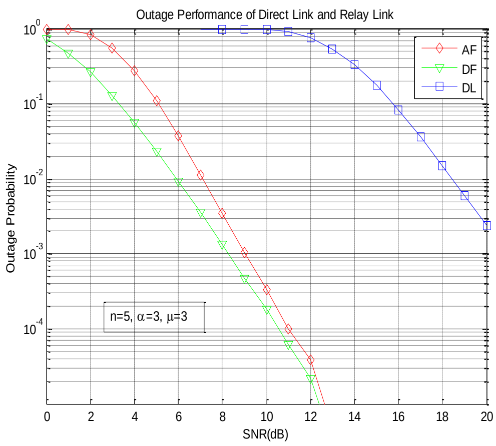

Fig. 6.9. Outage of two-hop relay system over α-μ fading for path loss exponent n=5, α=3, μ=3

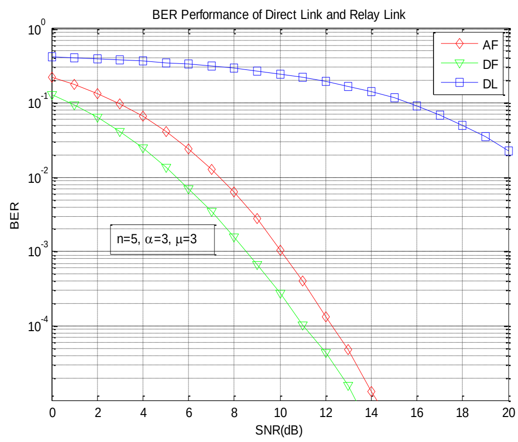

Fig. 6.10. BER of two-hop relay system over α-μ fading for path loss exponent n=5, α=3, μ=3

In fig.6.4 the BER performance for n=3, α=3, μ=2 is plotted. Here, it is noticed, at 10 dB SNR, the BER for DL is 6.0×10^{-2}, for AF is 6.5×10^{-3}, and for DF is 3.0×10^{-3}, showing that performance of DF is better among all. BER of DL again decreases in fig.6.6, due to increase in PL exponent.

In fig.6.7, and fig.6.9, the outage performance for path loss exponent n=3 and 5 respectively are shown keeping α and μ equal to 3. It is seen that at 10 dB SNR, the outage for DL is 3.0×10^{-2}, and 5.0×10^{-1} respectively in these plots. Thus, it is evident that with increase in PL exponent the outage performance of DL decreases, but the outage for AF and DF remains unperturbed. The BER performance for path loss exponent n = 3, and 5 are shown in fig.6.8, and fig.6.10, respectively, when α and μ is kept as equal to 3. It is observed that at 10 dB SNR, the BER for DL is 3.0×10^{-2}, and 1.7×10^{-1} respectively in these figures. The simulation results shows that BER performance for DL deteriorates with increase in PL exponent for obvious reasons, at the same time there is no significant effect on BER performance for AF and DF mode.

In this analysis outage probability and BER performance of two-hop relay based wireless system, have been analysed for amplify-and-forward (AF) mode, and decode-and-forward (DF) mode over α-μ fading channel. The performance of AF and DF mode is compared with direct link (DL) with help of simulation results. The effect of path loss exponent (n) parameter and α and μ parameters on outage and BER performance is brought out. It has been analyzed that with increase in path loss exponent, DL performance deteriorates. The relay based system outperforms the direct communication with large margin, and this difference increases with increase of path loss exponent. This analysis have been documented as a research paper ***"Comparative Performance of Two-hop Relay System over α-μ Fading Channel***", and is under revision in the Springer, The Institution of Engineers (India) B series. (IEIB-S-15-00483) for publication.

6.2 Power Optimization in Regenerative Relay-based Communication System:

6.2.1 Uniform Power Allocation:

Here we formulate the problem, which will be investigated through simulation. Scenario of a network is considered, in which individual node has upper bound of energy (i.e. limited power wireless terminal).

$$0 < P_s < P_s^{\max} \ \& \ \ 0 < P_r < P_r^{\max} \tag{6.31}$$

These terminals works in network of cellular type or works in unlicensed band. Total power radiated by network in this band should not exceed beyond specified level to avoid interfering other networks work in this band, else the total network is battery powered whose total power is known. Such types of network have upper bound on total power transmission. So transmitted power by source is given by

$$P_s = \lambda P_T; \qquad 0<\lambda<1 \tag{6.32}$$

Here P_T is total available power in this network. Transmitted power by supporting relay is given by

$$P_r = (1-\lambda) P_T \tag{6.33}$$

In traditional uniform power allocation, power is equally distributed between source (s) and relay (r), without taking channel state information (CSI) into account.

$$P_s = \frac{1}{2} P_T, \text{ and } \ P_r = \frac{1}{2} P_T \tag{6.34}$$

This is the simplest schemes and supporting relay placed at middle gives optimum capacity gain.

6.2.2 Optimum Power Allocation:

Optimum power allocation is a centralized power allocation technique in which source should have full CSI between all nodes prior to transmission. When each node operate in TDMA mode, practically it is feasible that channels are estimated by sending training sequence before the actual message transmission. When the source (s) transmits the training bits, all relay node can simultaneously estimate their source-to-relay CSI, due to broadcast nature of the wireless medium. Similarly when relay (r) transmits the training bits, the CSI of source-to-relay and relay-to-destination can be estimated at source and destination (d) respectively. (We assume that forward and backward channels between relay and destination are the same due to reciprocity. These transmission occur on the same frequency band and same coherence interval). However, CSI of relay-to-destination can be available at the source only through channel feedback.

In a slow fading environment, frequent training is not necessary. Hence, in this case we can neglect training period as compared to actual data transmission period. On the basis of CSI i.e. (γ_{sd}, γ_{sr}, γ_{rd}), source distributes the available power (P_T) between s and r. So, our primary objective is effectively utilizing the available power to boost γ_T at destination i.e.

$$\max_{P_s,\, P_r} \lambda_T = \min(\delta_{sr}, P_s, \delta_{rd}, P_r) \tag{6.35}$$

6.2.3 Power Optimization in Regenerative relay-based system over α-μ fading:

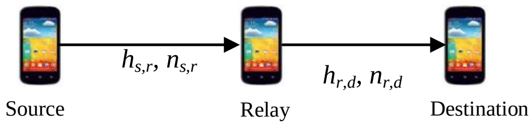

Fig.6.11. Two-hop wireless communication system with relay

Outage and BER performance for two-hop regenerative relay based system over α-μ fading channel is obtained by Monte-Carlo simulation. The communication system shown in fig.6.11 has been considered in this simulation. Outage probability and BER performance simulated results for power optimized and uniform power for different α and μ values are shown in fig. 6.12 to fig. 6.19. In these simulation 2000 bits have been considered for a particular α and μ combination. In direct link between source-destination BPSK modulation is used, whereas in relay link QPSK modulation is used to maintain the same data rate for honest performance comparison. In simulation we used the finding of

Waltz, et.al., (2006) who described that constrained non-linear optimisation or non-linear programming attempts to find a constrained minimum of a scalar function of scalar variables starting at an initial estimate.

In fig.6.12 the outage performance for α=1, μ=1 is shown. It is seen that at 15 dB SNR, the outage for uniform power case is higher as $\approx 10^{-0.6}$, and that for optimized power case is $\approx 10^{-0.7}$. In fig.6.14, α is increased to 2 and μ parameter is same as in fig.6.13, outage for uniform power case is 8.0×10^{-2}, and for optimized case is 5.0×10^{-2} thus we find an improvement in outage with increase in α value.

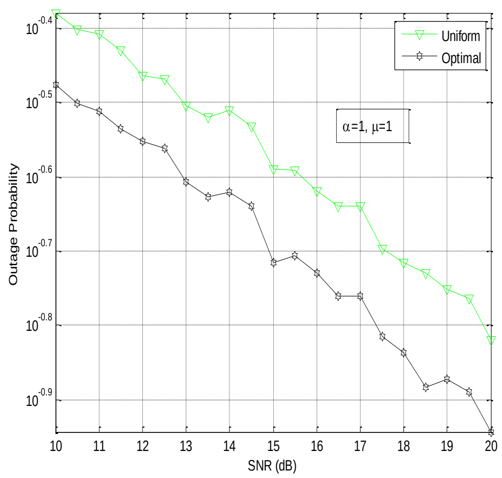

Fig. 6.12. Outage of power allocated two-hop regenerative relay over α-μ fading for α=1, μ=1

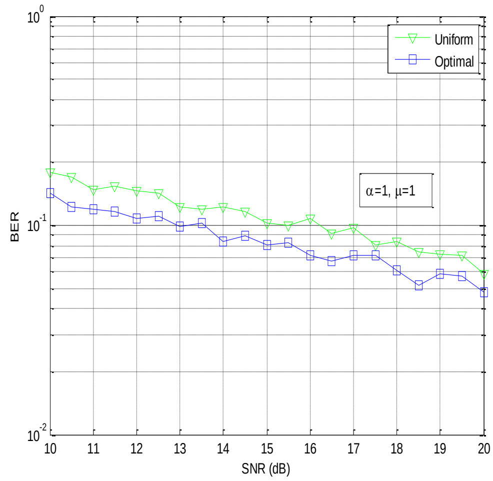

Fig. 6.13. BER of power allocated two-hop regenerative relay over α-μ fading for α=1, μ=1

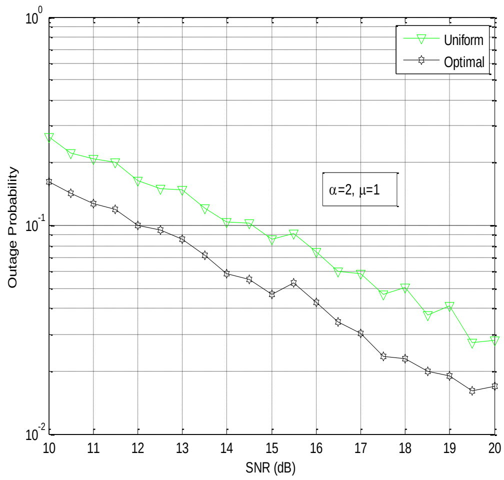

Fig. 6.14. Outage of power allocated two-hop regenerative relay over α-μ fading for α=2, μ=1

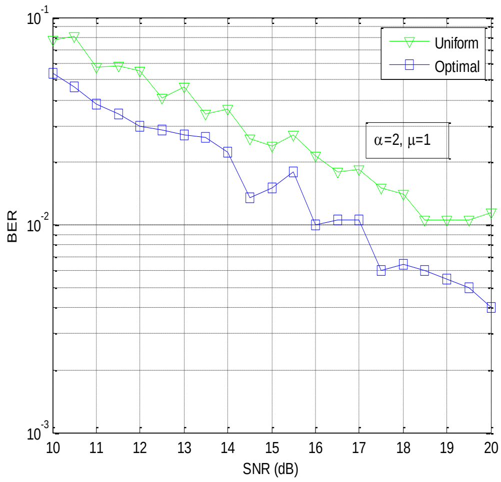

Fig. 6.15. BER of power allocated two-hop regenerative relay over α-μ fading for α=2, μ=1

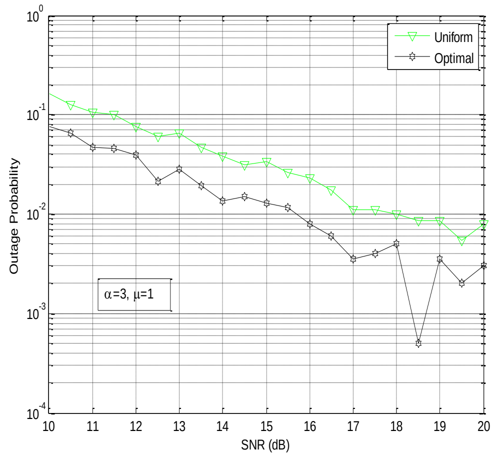

Fig. 6.16. Outage of power allocated two-hop regenerative relay over α-μ fading for α=3, μ=1

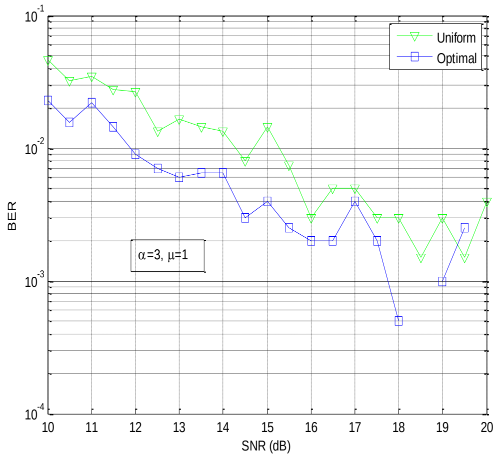

Fig. 6.17. BER of power allocated two-hop regenerative relay over α-μ fading for α=3, μ=1

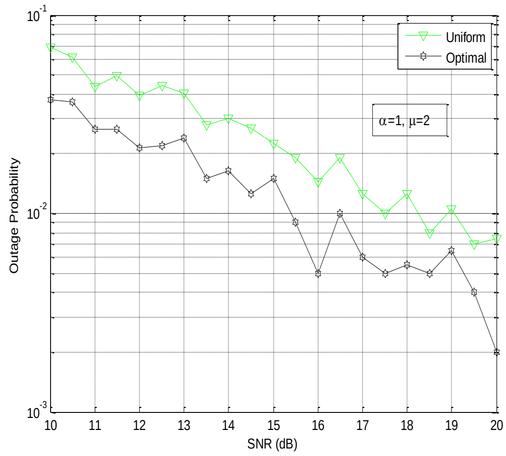

Fig.6.18. Outage of power allocated two-hop Regenerative relay over α-μ fading for α=1, μ=2

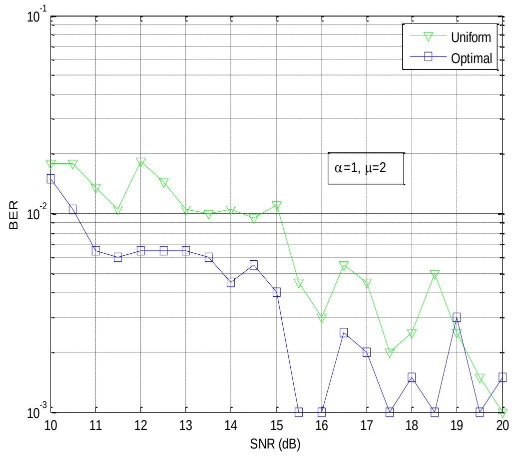

Fig. 6.19. BER of power allocated two-hop regenerative relay over α-μ fading for α=1, μ=2

Further improvement in outage is noticed in fig.6.16 with increase in α equal to 3 and μ=1, here it is noticed, at 15 dB SNR, the outage for uniform power case is 2.3×10^{-2}, and for optimized power case is 1.3×10^{-2}. In fig.6.18 with α=1, μ=2, the outage for uniform power case is observed as 2.2×10^{-2}, and for optimized power case is 1.7×10^{-2}.

In fig.6.13 the BER performance for α & μ equal to 1 is shown. It is seen that at 15 dB SNR, the BER for uniform power case is higher as $\approx10^{-1}$, and that for optimized power case is 8.0×10^{-2}. In fig.6.15, the BER performance for α=2 and μ=1, it is seen that at 15 dB SNR, the BER for uniform power case is 2.5×10^{-2}, and that for optimized power case is 1.6×10^{-2}. Fig.6.17 shows the BER for α=3 and μ=1, it is found that at 15 dB SNR, the

BER for uniform power case is 1.5×10^{-2}, and that for optimized power case is 4.0×10^{-3}. In fig. 6.19, for α=1, μ=2, BER for uniform power case is noted as 1.2×10^{-2}, and that for optimized power is 4.0×10^{-3}.

In this analysis outage probability and BER performance of two-hop relay based wireless system, have been analysed for uniform power and optimized power condition is analyzed over α-μ fading channel. The performance of regenerative relay system is compared with simulation results for uniform power and power optimized scenario. It has been analyzed that with increase in α and μ parameters, BER and outage performance improves. The optimized power relay based system outperforms the uniform power system. The results of this analysis is published in a research paper, ***"Optimal Power Allocation for Regenerative Relay over α-μ Fading Channel***" in International Journal of Computer and Communication Technologies (IJCCTS), ISSN 2278-9723, vol.3, no.11, Sep.2015, pp.761-766.

6.3 Multi-hop Serial-Relay Communication System over *α-μ* fading:

The process of relaying on dual-hop can be extended to multi-hop communication by allowing the signal to traverse through the multiple intermediate nodes, as shown in fig.6.20. Adam Helmut et. al.(2009), presented a complete system design of multi-hop-aware cooperative relaying and investigated different relay selection policies for it. Multi-hop relay networks significantly improve the communication coverage of cellular and ad hoc networks without spending extra network resources such as bandwidth and power. In a multi-hop serial relay-network wireless communication system shown in fig.6.20, the signal is transmitted as broadcast through the serial relay network placed between source (*s*) and destination (*d*).

$$\text{The relay network consists of } n \text{ serial relays} = \left(\{r_k\}_{k=1}^{n}\right)$$

In the communication system shown in fig.6.20, s transmits to relay 1 ($s \rightarrow r_1$) in first time-slot, in second time slot relay 1 transmits to relay 2 ($r_1 \rightarrow r_2$) and so on, and finally in $(n+1)^{th}$ time slot relay n transmits to d ($r_n \rightarrow d$). PDF of the SNR for α-μ distributed channel is defined by (2).

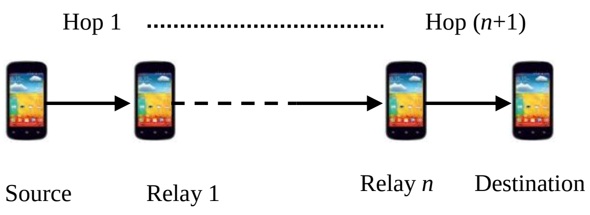

Fig.6.20. Multi-hop serial relay network communication system

6.3.1 Multi-hop Serial-Relay Amplify and Forward (AF) Protocol:

During AF protocol in the communication system shown in fig.6.20, the relays receives the signal, amplifies it and then transmits to next stage. Assuming that source transmitting signal x, which is received by 1^{st} relay as

$$y_{s,r_1} = h_{s,r_1} x + n_{s,r_1} \tag{6.36}$$

The received signal is scaled by a factor G_1, gain of relay 1. Thus, the signal received by 1^{st} relay is forwarded to 2^{nd} relay, and is received at 2^{nd} relay as

$$y_{r_1,r_2} = G_1 h_{r_1,r_2} y_{s,r_1} + n_{r_1,r_2} \tag{6.37}$$

where

h_{r_i,r_j}, n_{r_i,r_j} = fading amplitude defined by eq.(1) and Additive White Gaussian Noise (AWGN) with variance σ^2 respectively for the (node r_i $\rightarrow$ node r_j) link of $\alpha-\mu$ wireless channel.

The received signal at relay 2 simplified from eq.(6.37) as

$$y_{r_1,r_2} = h_{s,r_1} h_{r_1,r_2} G_1 x + h_{r_1,r_2} G_1 n_{s,r_1} + n_{r_1,r_2} \tag{6.38}$$

Thus, SNR at receiver of relay 2, from eq.(6.38) will be

$$\text{SNR}=\frac{(h_{s,r_1} h_{r_1,r_2} G_1)^2}{(h_{r_1,r_2} G_1)^2 \sigma^2 + \sigma^2} \tag{6.39}$$

$$\text{or} \qquad \text{SNR} = \frac{h_{s,r_1}^2 h_{r_1,r_2}^2}{h_{r_1,r_2}^2 \sigma^2 + \sigma^2 / G_1^2} = \frac{\frac{h_{s,r_1}^2}{\sigma^2} \frac{h_{r_1,r_2}^2}{\sigma^2}}{\frac{h_{r_1,r_2}^2}{\sigma^2} + \frac{1}{G_1^2 \sigma^2}} \tag{6.40}$$

$$\text{Let, SNR} = \gamma_i = \frac{h_i^2}{\sigma^2}, \text{ and } G_1^2 = \frac{1}{h_{s,r_1}^2 + \sigma^2} \tag{6.41}$$

Hence SNR for AF at receiver of relay 2 from eq.(6.40) & (6.41) will be

$$\gamma_{AF} = \frac{\gamma_{s,r_1}\gamma_{r_1,r_2}}{\gamma_{s,r_1} + \gamma_{r_1,r_2} + 1} \approx \frac{\gamma_{s,r_1}\gamma_{r_1,r_2}}{\gamma_{s,r_1} + \gamma_{r_1,r_2}} = \frac{1}{\frac{1}{\gamma_{s,r_1}} + \frac{1}{\gamma_{r_1,r_2}}} \tag{6.42}$$

Since $\left(\gamma_{s,r_1} + \gamma_{r_1,r_2}\right) \gg 1$

Similar to eq.(6.38), the received signal at relay 3 will be

$$y_{r_2,r_3} = h_{s,r_1} h_{r_1,r_2} h_{r_2,r_3} G_1 G_2 x + h_{r_1,r_2} h_{r_2,r_3} G_1 G_2 n_{s,r_1} + h_{r_2,r_3} G_2 n_{r_1,r_2} + n_{r_2,r_3} \tag{6.43}$$

Similarly as in eq.(6.43), the received signal at relay k will be

$$\begin{aligned} y_{r_{k-1},r_k} &= h_{s,r_1} h_{r_1,r_2} h_{r_2,r_3} \ldots\ldots h_{r_{k-1},r_k} G_1 G_2 \ldots\ldots G_{k-1} x \ + \ h_{r_1,r_2} h_{r_2,r_3} \ldots\ldots\ldots\ldots h_{r_{k-1},r_k} G_1 G_2 \ldots\ldots G_{k-1} n_{s,r_1} \\ &\quad + h_{r_2,r_3} \ldots\ldots\ldots\ldots h_{r_{k-1},r_k} G_2 \ldots\ldots G_{k-1} n_{r_1,r_2} \ + \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ + \ n_{r_{k-1},r_k} \end{aligned} \tag{6.44}$$

Finally the received signal for multi-path serial-relays at destination d will be

$$\begin{aligned} y_{r_n,d} &= h_{s,r_1} h_{r_1,r_2} h_{r_2,r_3} \ldots\ldots h_{r_n,r_d} G_1 G_2 \ldots\ldots G_n x + \ h_{r_1,r_2} h_{r_2,r_3} \ldots\ldots\ldots\ldots h_{r_n,r_d} G_1 G_2 \ldots\ldots G_n n_{s,r_1} \\ &\quad + h_{r_2,r_3} h_{r_3,r_4} \ldots\ldots\ldots\ldots h_{r_n,r_d} G_2 G_3 \ldots\ldots G_n n_{r_1,r_2} \ + \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\quad + h_{r_{k-1},r_k} h_{r_k,r_{k+1}} \ldots\ldots h_{r_n,r_d} G_{k-1} G_k \ldots\ldots G_n n_{r_{k-2},r_{k-1}} \ + \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + n_{r_n,r_d} \end{aligned} \tag{6.45}$$

Following steps of eq.(6.39) to (6.42) SNR at receiver of d for AF protocol multi-path serial-relays can be generalised for n relays, and will be as per eq. (1) of Yilmaz, Tabassum & Alouini, (2014) and eq.(4) of Yilmaz, Kucur & Alouini, (2010) given as

$$\gamma_{AF} = \frac{1}{\frac{1}{\gamma_{s,r_1}} + \frac{1}{\gamma_{r_1,r_2}} + \ldots\ldots + \frac{1}{\gamma_{r_{n-1},r_n}} + \frac{1}{\gamma_{r_n,d}}} \tag{6.46}$$

From the SNR for AF protocol obtained in eq.(6.46), the PDF of SNR (γ) for AF protocol over α-μ fading may be obtained with help of eq.(6.47) reproduced here from eq.(3.56):

$$f_{\gamma_{ij}}(\gamma) = \frac{\alpha\ \mu^{\mu}\ \gamma^{\frac{\alpha\mu}{2}-1}}{2\ \Gamma(\mu)\ \overline{\gamma}_{ij}^{\ \alpha\mu/2}}\quad e^{-\mu\left(\frac{\gamma}{\overline{\gamma}_{ij}}\right)^{\alpha/2}} \tag{6.47}$$

here, $i \in \{s, r_k\},\ j \in \{r_k, d\}$.

Thereafter the outage and BER for multi-hop serial relay-network system with AF protocol may be given as

$$P_{out}\big|_{AF} = \int_0^{\gamma_{th}} f_{\gamma_{AF}}(\gamma)\,d\gamma \tag{6.48}$$

$$BER\left(P_e\big|_{AF}\right) = \int_0^{\infty} Q\left(a\sqrt{\gamma}\right) f_{\gamma_{AF}}(\gamma)\ d\gamma \tag{6.49}$$

where γ_{th} is threshold SNR, a is a constant given by eq. (5.1) of Simon and Alouini, (2005) and depends on modulation & detection combination.

The threshold SNR value of relaying for (n+1) hops is defined from

since, $R = \dfrac{1}{n+1}\log_2(1+SNR) \ \Rightarrow\ (n+1)R = \log_2(1+SNR)$

or $2^{(n+1)R} = 1 + SNR \ \Rightarrow$ threshold $SNR = \gamma_{\text{th}} = 2^{n+1} - 1$ (6.50)

Here R is assumed to be unity for numerical evaluation. Each serial relay utilizes one time slot out of (n+1) time slots, hence spectrum efficiency of such systems become 1/(n+1) part of available spectrum. So, modulation parameter $M = 2^{n+1}$ and $\gamma_{th} = (2^{n+1} - 1)$ has been chosen in half duplex (HD) for faithful comparison between different numbers of cooperative relay nodes, whereas in full duplex (FD) M is taken as 2 and γ_{th} =1.

The *HD* system allows two-way communication by using same channel for both transmission and reception. Thus at any given time, the user can only either transmit or receive information. Whereas *FD* system allows simultaneous two-way communication. Here transmission and reception are on two different channels. Relay operation in *HD* mode, requires additional time slot to separate the incoming and outgoing signals hence consumes extra resource (bandwidth). *FD* mode in single operating frequency does not require extra time-slot, hence there is no bandwidth expansion. Relay operating in this mode have separate set of transmitting and receiving antennas but still suffer from loop interference due to insufficient electrical isolation between transmitting and receiving set antennas.

6.3.2 Multi-hop Serial-Relay Decode and Forward (DF) Protocol:

The decode and forward (DF) is a digital and regenerative scheme, where relay receives the signal, decodes it and after encoding retransmit it to the destination. Noise does not propagate, because the noise will not be amplified and is excluded by the decoding process. The signal received at 1st relay is given by eq.(6.36), thereafter signal is decoded at 1st relay and encoded.

Let the encoded signal be $\hat{x}_1$.

This we obtain by maximum likelihood detector, when signal x_1 is estimated by relay 1.

$$\hat{x}_1 = \arg\min_x \left| y_{s,r_1} - h_{s,r_1} x \right|^2 \tag{6.51}$$

Hence, the signal received in DF, at relay 2 will be

$$y_{r_1,r_2} = h_{r_1,r_2}\hat{x}_1 + n_{r_1,r_2} \tag{6.52}$$

The SNR for 1st hop and 2nd hop i.e. input and output hops of 1st relay is defined as

$$\text{For } 1^{st} \text{ hop,} \quad \gamma_{s,r_1} = \frac{h_{s,r_1}^2 P_s}{\sigma^2} \tag{6.53}$$

$$\text{For } 2^{nd} \text{ hop,} \quad \gamma_{r_1,r_2} = \frac{h_{r_1,r_2}^2 P_{r_1}}{\sigma^2} \tag{6.54}$$

P_s and P_{r1} are the transmit power at source and 1st relay respectively and have been normalized to unity. Similar to eq. (6.53), (6.54), SNR for all other hops may be given as

$$\text{For } k^{th} \text{ hop,} \quad \gamma_{r_{k-1},r_k} = \frac{h_{r_{k-1},r_k}^2 P_{r_{k-1}}}{\sigma^2} \tag{6.55}$$

$$\text{For last i.e. } (n+1)^{th} \text{ hop,} \quad \gamma_{r_n,d} = \frac{h_{r_n,d}^2 P_{r_n}}{\sigma^2} \tag{6.56}$$

The, SNR in DF protocol with multi-hop serial-relay system, at receiver of d will be

$$\gamma_{DF} = \min\left(\gamma_{s,r_1},\ \gamma_{r_1,r_2},....,\gamma_{r_{k-1},r_k},....,\gamma_{r_{n-1},r_n},\gamma_{r_n,d}\right) \tag{6.57}$$

From the SNR for DF obtained in eq.(6.57), PDF of SNR may be obtained with help of eq. (6.47). Thereafter the outage and BER for DF may be given as

$$P_{out}\big|_{DF} = \int_0^{\gamma_{th}} f_{\gamma_{DF}}(\gamma)\,d\gamma \tag{6.58}$$

$$BER\left(P_e\big|_{DF}\right) = \int_0^{\infty} Q\left(a\sqrt{\gamma}\right) f_{\gamma_{DF}}(\gamma)\ d\gamma \tag{6.59}$$

6.3.3 Performance Analysis of Multi-hop Serial-relay AF and DF protocol over *α-μ* fading:

Outage and BER performance for multi-hop relay-network communication system over α-μ fading channel is obtained by Monte-Carlo simulation for half-duplex (*HD*) and full-duplex (*FD*) systems. The communication system shown in fig.2 has been considered in this simulation. Outage probability and BER performance for amplitude-and-forward protocol and decode-and-forward protocol simulated results considering various number of serial relays for half-duplex are shown in fig. 6.21 to fig.6.26 and for full-duplex are shown in fig. 6.27 to fig.6.32. In these simulation 100000 bits have been considered for a particular α, μ, and relay combination.

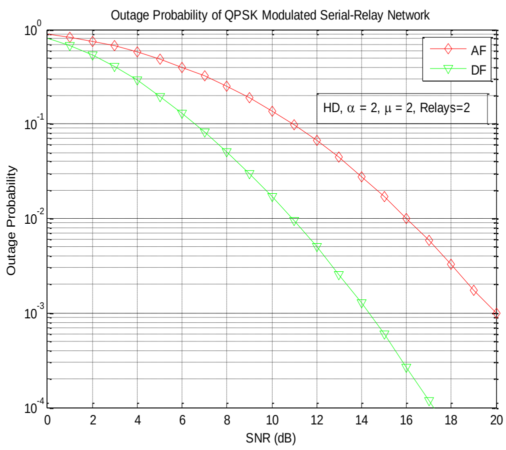

Fig. 6.21. Outage of Multi-Hop Half-Duplex serial Relay-Network over α-μ fading channel for α=2, μ=2, Relays=2.

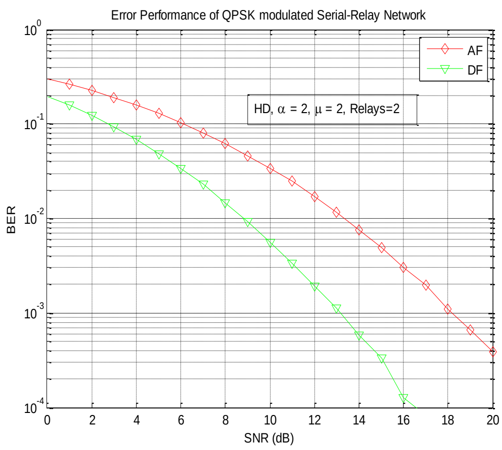

Fig. 6.22. BER of Multi-Hop Half-Duplex serial Relay-Network over α-μ fading channel for α=2, μ=2, Relays=2.

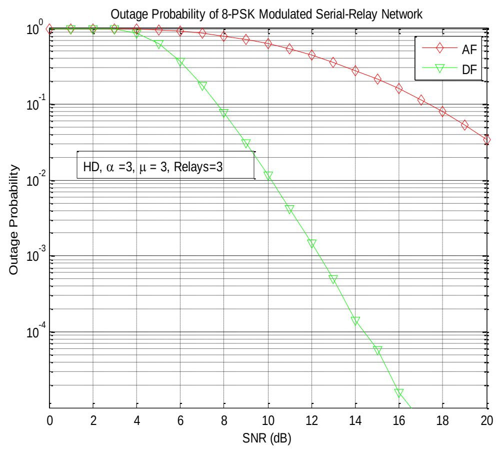

Fig. 6.23. Outage of Multi-Hop Half-Duplex serial Relay-Network over α-μ fading channel for α=3, μ=3, Relays=3.

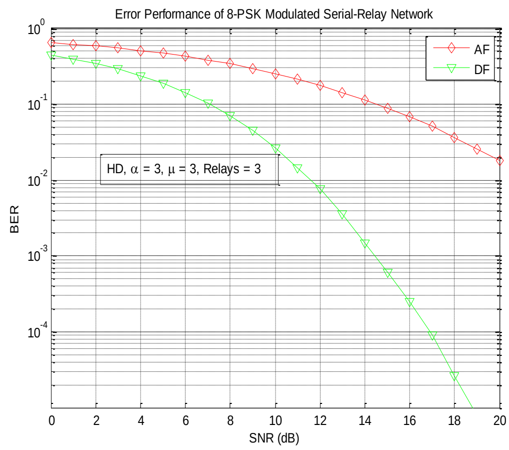

Fig. 6.24. BER of Multi-Hop Half-Duplex serial Relay-Network over α-μ fading channel for α=3, μ=3, Relays=3.

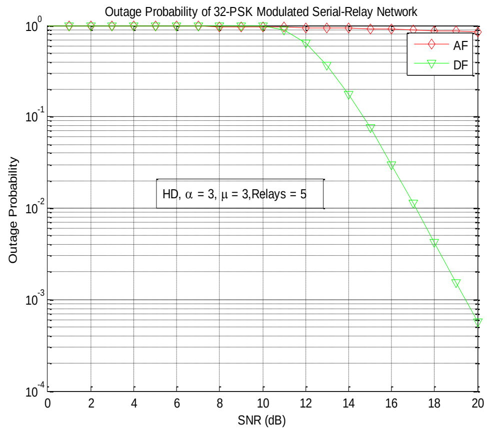

Fig. 6.25. Outage of Multi-Hop Half-Duplex serial Relay-Network over α-μ fading channel for α=3, μ=3, Relays=5.

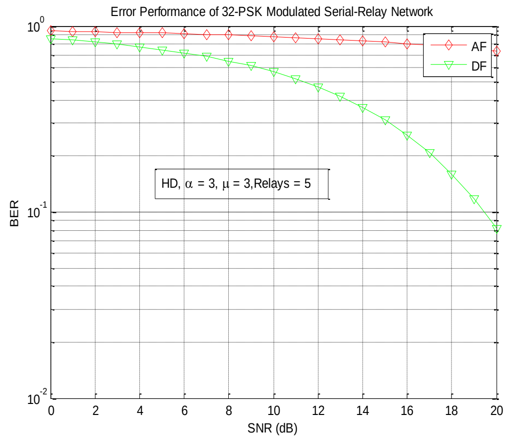

Fig. 6.26. BER of Multi-Hop Half-Duplex serial Relay-Network over α-μ fading channel for α=3, μ=3, Relays=5.

Half-duplex simulation results are shown in fig.6.21 to fig.6.26. In fig.6.21 the outage performance for serial relays=2, and α=2, μ=2 is shown. It is seen that at 10 dB SNR, the outage for AF protocol is 1.5×10^{-1}, and that for DF is 1.8×10^{-2}. In fig.6.22, we find the BER for same system as 3.4×10^{-2} for AF, and 5.5×10^{-3} for DF. In fig.6.23 and fig.6.24 all the parameters i.e. serial relays, α and μ are increased to 3, we notice that at 10 dB SNR, for AF and DF outage as 6.5×10^{-1} and 1.2×10^{-2}, and BER as 2.5×10^{-1} and 2.8×10^{-2} respectively.

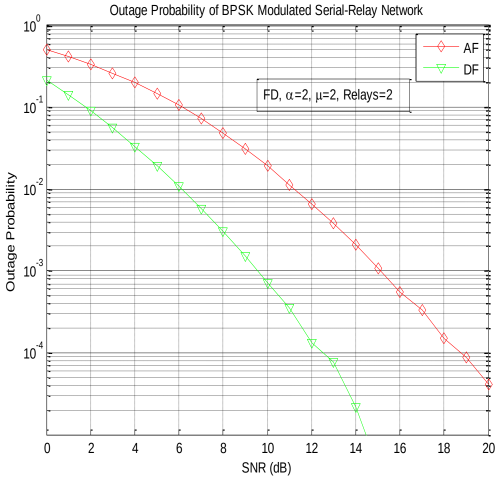

Fig. 6.27. Outage of Multi-Hop Full-Duplex serial Relay-Network over α-μ fading channel for α=2, μ=2, Relays=2.

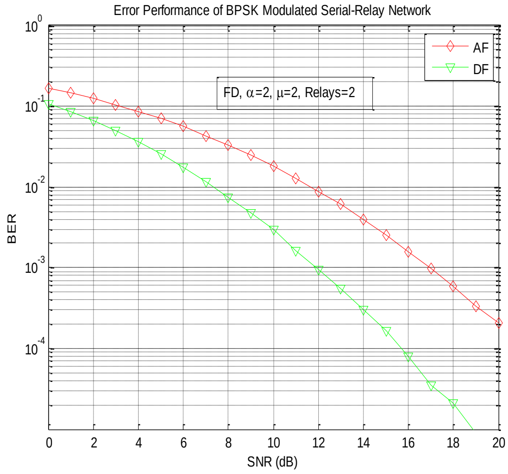

Fig. 6.28. BER of Multi-Hop Full-Duplex serial Relay-Network over α-μ fading channel for α=2, μ=2, Relays=2.

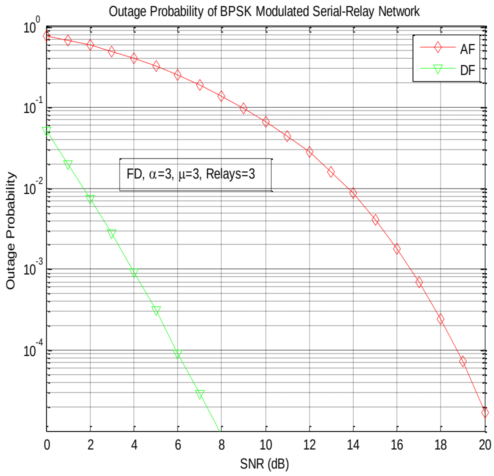

Fig. 6.29. Outage of Multi-Hop Full-Duplex serial Relay-Network over α-μ fading channel for α=3, μ=3, Relays=3.

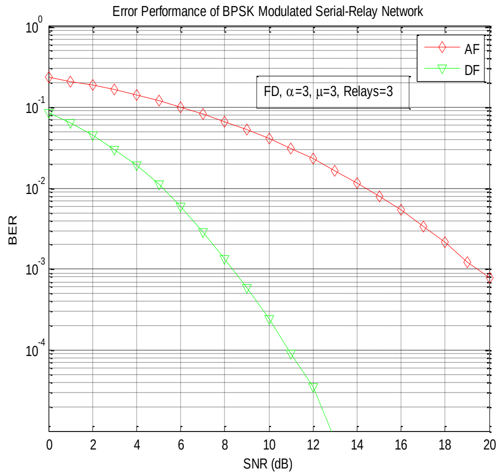

Fig. 6.30. BER of Multi-Hop Full-Duplex serial Relay-Network over α-μ fading channel for α=3, μ=3, Relays=3.

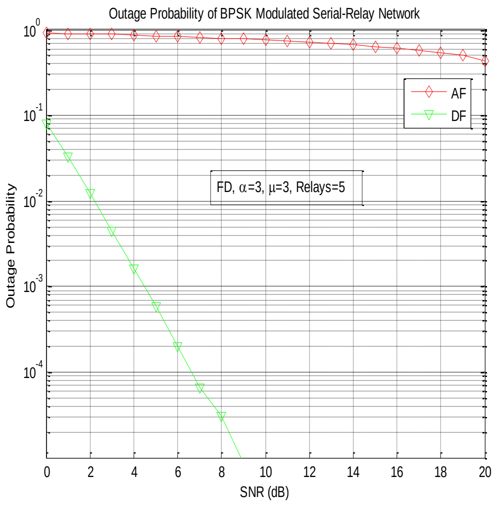

Fig. 6.31 Outage of Multi-Hop Full-Duplex serial Relay-Network over α-μ fading channel for α=3, μ=3, Relays=5.

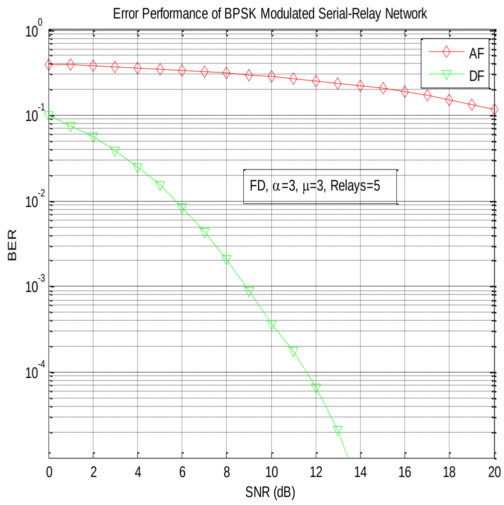

Fig. 6.32. BER of Multi-Hop Full-Duplex serial Relay-Network over α-μ fading channel for α=3, μ=3, Relays=5.

In fig.6.25 and fig. 6.26 serial relays are again increased to 5 and α and μ parameters are maintained as 3, we notice that at 10 dB SNR for AF and DF outage is same approximately 9.5×10^{-1}, and BER as 8.5×10^{-1} and 5.7×10^{-1} for AF and DF respectively. Full-duplex simulation results are shown in fig.6.27 to fig.6.32. In fig.6.27 the outage performance for serial relays=2, and α=2, μ=2 is shown. It is seen that at 10 dB SNR, the outage for AF protocol is 2×10^{-2}, and that for DF is 7×10^{-4}. In fig.6.28, we find the BER for same system as 2×10^{-2} for AF, and 3×10^{-3} for DF. In fig.6.29 and fig.6.30 all the parameters i.e. serial relays, α and μ are increased to 3, we notice that at 10 dB SNR, outage for AF as 6×10^{-2} and at 6 dB SNR outage for DF as 8×10^{-5}, and BER as 4×10^{-2} and 2.5×10^{-4} respectively at 10 dB SNR. In fig.6.31 and fig.6.32 serial relays are again increased to 5 and α and μ parameters are maintained as 3, we notice that at 10 dB SNR for AF outage is 8×10^{-1} and at 6 dB SNR for DF outage is 2×10^{-4}, and BER at 10 dB SNR as 3×10^{-1} and 3.5×10^{-4} for AF and DF respectively.

The simulation results shows that as expected outage and BER performance for DF protocol is always better than AF protocol. We also observe that as the number of serial relays are increased, outage and BER for AF and DF deteriorates. Further deterioration in AF is more comparative to DF, because in AF with increase in number of relays, noise also gets amplified accordingly. We also observe that performance of full-duplex is better than half-duplex for similar configuration, due to proper utilization of spectrum. Thus we observe that with increase in relays communication of longer range is achieved but at the cost of deterioration in outage and BER performance.

In this analysis outage probability and BER performance of multi-hop relay-network wireless communication system have been analysed for amplify-and-forward (AF) protocol, and decode-and-forward (DF) protocol over α-μ fading channel. The outage and BER performance AF protocol is compared with DF protocol with help of simulation results. The effect of number of serial relays on outage and BER performance is brought out. It has been analyzed that with increase in number of serial relays, longer range communication is achieved on the price of spectrum efficiency for half-duplex. For same spectrum efficiency, full-duplex relays outperform half-duplex relays. As number of relays increases noise is amplified and forwarded in next hop for the case of AF relay and error is forwarded in next hop for the case of DF relay hence performance deteriorates. Further, DF protocol outperforms AF protocol systems with large margin, and this difference increases with increase in number of serial relays. Full-duplex performance is much better than half-duplex for same system configuration. This analysis results are published as a research paper titled, ***"Performance Analysis of Multi-Hop Relay-Network over α-μ Fading Channel***", in International Journal of Research in Electronics and Communication Technology (IJRECT), ISSN 2348-9065, vol.2, iss.3, Jul-Sep 2015, pp.22-28.

6.4 Performance Analysis of Cooperative Communication System over *α-μ* fading:

In a two-hop multi-relay wireless cooperative communication system shown in fig.6.33., the signal is transmitted as broadcast through the multiple -relay network placed between source and destination. In the communication system shown in fig.6.33,

The relay network consists of n relays $= \left(\{r_k\}_{k=1}^{n}\right)$

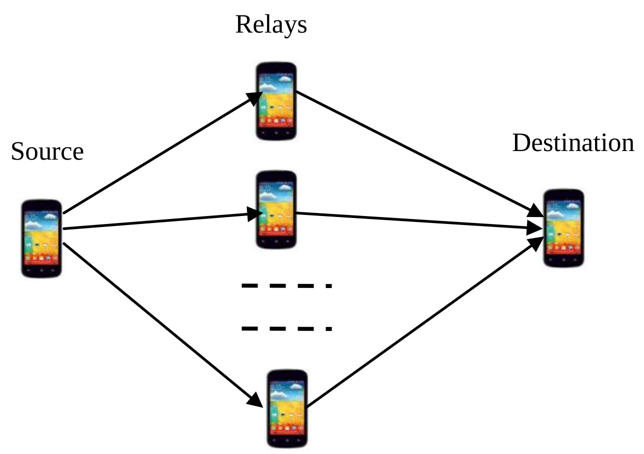

Fig.6.33. Cooperative Communication system with parallel multiple-relays

6.4.1 Two-hop Amplify and Forward (AF) Cooperative Communication

Due to half-duplex nature of relays, s transmits to all n relays i.e. (r_1, r_2,……r_k,……r_n) in first time-slot. Then in second time-slot r_1 will transmit to d, in 3rd time-slot r_2 transmits to d and so on. In $(n+1)^{th}$ time slot r_n will transmit to d. Direct link available between source and destination has not been taken into consideration.

SNR for two-hop AF relay system obtained in eq. (6.10) as

$$\gamma_{AF} = \frac{1}{\dfrac{1}{\gamma_{s,r}} + \dfrac{1}{\gamma_{r,d}}}$$

Therefore SNR for AF, k^{th} relay link from eq. (6.10) may be given as

$$^{k}\gamma_{AF} = \frac{1}{\dfrac{1}{\gamma_{s,r_k}} + \dfrac{1}{\gamma_{r_k,d}}} \tag{6.60}$$

Thus, in AF protocol two-hop system, multi-copies of signal x is received by the destination through multiple relays. These signals may be combined with any of the different combining techniques. The optimum combining technique that maximizes the overall signal to noise ratio is called the maximal ratio combiner (MRC). SNR for MRC will be

$$\gamma_{AF}^{MRC} = {}^{1}\gamma_{AF} + {}^{2}\gamma_{AF} + \ldots\ldots + {}^{k}\gamma_{AF} + \ldots\ldots + {}^{n}\gamma_{AF} \tag{6.61}$$

From the SNR for MRC-AF obtained in eq.(6.61), the PDF of SNR over α-μ channel may be obtained with help of eq.(6.47) reproduced as below

$$f_{\gamma_{ij}}(\gamma) = \frac{\alpha\ \mu^{\mu}\ \gamma^{\frac{\alpha\mu}{2}-1}}{2\ \Gamma(\mu)\ \overline{\gamma}_{ij}^{\ \alpha\mu/2}} \quad e^{-\mu\left(\frac{\gamma}{\overline{\gamma}_{ij}}\right)^{\alpha/2}}$$

here, $i \in \{s, r_k\}$, $j \in \{r_k, d\}$, $i\ \&\ j \neq r_k$ simultaneously.

Thereafter the outage and BER for MRC - AF may be given as

$$P_{out}\Big|_{AF}^{MRC} = \int_{0}^{\gamma_{th}} f_{\gamma_{AF}^{MRC}}(\gamma)\,d\gamma \tag{6.62}$$

$$BER\left(P_e\Big|_{AF}^{MRC}\right) = \int_{0}^{\infty} Q\left(a\sqrt{\gamma}\right) f_{\gamma_{AF}^{MRC}}(\gamma)\; d\gamma \tag{6.63}$$

where γ_{th} is threshold SNR defined by eq.(6.50). Modulation parameter $M = 2^{n+1}$ and $\gamma_{th} = (2^{n+1} - 1)$ has been chosen in half duplex (HD) for faithful comparison between different numbers of cooperative relay nodes, whereas in full duplex (FD) M is taken as 2 and γ_{th} =1. Value of a in eq. (6.63) is a constant given by eq. (5.1) of Simon and Alouini, 2005, and depends on modulation & detection combination.

In place of MRC if selection combining (SC) is used which is based on principle of selecting the best signal among all the signals received from different branches at the destination, then the SNR for SC will be

$$\gamma_{AF}^{SC} = \max\left({}^{1}\gamma_{AF}, {}^{2}\gamma_{AF}, ..., {}^{k}\gamma_{AF}, ..., {}^{n}\gamma_{AF}\right) \tag{6.64}$$

Now from the SNR for SC obtained in eq.(6.64), similarly as above PDF of SNR over α-μ channel may be obtained with help of eq.(6.47). Thereafter the outage and BER for SC - AF may be given as

$$P_{out}\Big|_{AF}^{SC} = \int_{0}^{\gamma_{th}} f_{\gamma_{AF}^{SC}}(\gamma)\,d\gamma \tag{6.65}$$

$$BER\left(P_e\Big|_{AF}^{SC}\right) = \int_{0}^{\infty} Q\left(a\sqrt{\gamma}\right) f_{\gamma_{AF}^{SC}}(\gamma)\; d\gamma \tag{6.66}$$

6.4.2 Two-hop Decode and Forward (DF) Cooperative Communication:

The decode and forward (DF) is a digital & regenerative scheme, where relay receives the signal, decodes it and after encoding retransmit it to the destination. Noise does not propagate, because noise is not amplified as it is excluded by the decoding process. In DF processing time is higher causing delay, hence DF is not suitable for delay sensitive signals. The signal received at k^{th} relay from source is given by eq.(6.67),

$$y_{s,r_k} = h_{s,r_k} x + n_{s,r_k} \tag{6.67}$$

the signal is decoded at k^{th} relay and encoded.

Let the encoded signal be $\hat{x}_k$.

This will be obtained by maximum likelihood detector, when signal x_k is estimated by relay.

$$\hat{x}_k = \arg\min_x \left| y_{s,r_k} - h_{s,r_k} x \right|^2 \tag{6.68}$$

Hence, the signal received in DF, at destination will be

$$y_{r_k,d} = h_{r_k,d} \hat{x}_k + n_{r_k,d} \tag{6.69}$$

The SNR for both hops for k^{th} relay is defined as below

$$\gamma_{s,r_k} = \frac{h_{s,r_k}^2 P_s}{\sigma^2} = \delta_{s,r_k} P_s \tag{6.70}$$

$$\gamma_{r_k,d} = \frac{h_{r_k,d}^2 P_{r_k}}{\sigma^2} = \delta_{r_k,d} P_{r_k} \tag{6.71}$$

where $\delta_{s,r_k} = \frac{h_{s,r_k}^2}{\sigma^2}$, and $\delta_{r_k,d} = \frac{h_{r_k,d}^2}{\sigma^2}$, P_s and P_{r_k} are the transmit power at source and k^{th} relay respectively.

The, SNR at destination for k^{th} relay link in DF will be

$${}^{k}\gamma_{DF} = \min\left(\gamma_{s,r_k},\ \gamma_{r_k,d}\right) \tag{6.72}$$

In DF, multi-copies of signal will be received by the destination through multiple relays. When these signals are combined by MRC techniques, which maximizes the overall signal to noise ratio. The SNR for MRC-DF will be

$$\gamma_{DF}^{MRC} = {}^{1}\gamma_{DF} + {}^{2}\gamma_{DF} + \ldots\ldots + {}^{k}\gamma_{DF} + \ldots\ldots + {}^{n}\gamma_{DF} \tag{6.73}$$

From the SNR for MRC-DF obtained in eq.(6.73), PDF of SNR over α-μ channel may be obtained with help of eq.(6.47). Thereafter the outage and BER for MRC - DF may be given as

$$P_{out}\Big|_{DF}^{MRC} = \int_{0}^{\gamma_{th}} f_{\gamma_{DF}^{MRC}}\left(\gamma\right) d\gamma \tag{6.74}$$

$$BER\left(P_e\Big|_{DF}^{MRC}\right) = \int_{0}^{\infty} Q\left(a\sqrt{\gamma}\right) f_{\gamma_{DF}^{MRC}}\left(\gamma\right)\, d\gamma \tag{6.75}$$

Instead of MRC-DF if SC is used which is based on principle of selecting the best signal among all the signals received from different braches at the destination, then the SNR for SC-DF will be

$$\gamma_{DF}^{SC} = \max\left({}^{1}\gamma_{DF}, {}^{2}\gamma_{DF}, ..., {}^{k}\gamma_{DF}, ..., {}^{n}\gamma_{DF}\right) \tag{6.76}$$

Now from the SNR for SC-DF obtained in eq.(6.76), similarly as above PDF of SNR over α-μ channel may be obtained with help of eq.(6.47). Thereafter the outage and BER for SC - DF may be given as

$$P_{out}\Big|_{DF}^{SC} = \int_{0}^{\gamma_{th}} f_{\gamma_{DF}^{SC}}\left(\gamma\right) d\gamma \tag{6.77}$$

$$BER\left(P_e\Big|_{DF}^{SC}\right) = \int_{0}^{\infty} Q\left(a\sqrt{\gamma}\right) f_{\gamma_{DF}^{SC}}\left(\gamma\right)\, d\gamma \tag{6.78}$$

6.4.3 Performance Analysis of two-hop parallel multi-relay AF & DF cooperative communication over α-μ channel:

Outage and BER performance for cooperative communication system with multiple parallel relays, over α-μ fading channel is obtained by Monte-Carlo simulation. The communication system shown in fig.6.33 has been considered in this simulation. Outage probability and BER performance simulated results for different number of parallel relays are shown in fig. 6.34 to fig.6.41 for amplitude-and-forward protocol and decode-and-forward protocol. In these simulation 100000 bits have been considered for a particular α, μ, and relay combination. The direct link between source-destination is not considered in this simulation.

In fig.6.34 the outage performance for HD relaying for 2 parallel relays, and α=2, μ=2 is shown. It is seen that at 5 dB SNR, the outage for MRC-AF is 8.0×10^{-5} but for MRC-DF is 4.0×10^{-5}, and for SC-AF is 6×10^{-3} whereas for SC-DF is 2×10^{-3} . In fig.6.35, we find the BER for same system at 6 dB SNR as 1.5×10^{-4} for MRC-AF, 5.0×10^{-5} for MRC-DF and for SC–AF as 3.0×10^{-3} whereas for SC–DF as 1.2×10^{-3}. In fig.6.36 the outage for FD relaying for 2 parallel relays, α=2, and μ=2 is shown. It is seen that at 1 dB SNR, the outage for MRC-AF is 2.8×10^{-5} but for MRC-DF is 1.5×10^{-5}, and for SC-AF is 2.3×10^{-3} whereas for SC-DF is 8×10^{-4} . In fig.6.37, we find the BER for same system at 6 dB SNR as 8×10^{-5} for MRC-AF, 3.5×10^{-5} for MRC-DF and for SC–AF as 1.5×10^{-3} whereas for SC–DF as 6.5×10^{-4} .

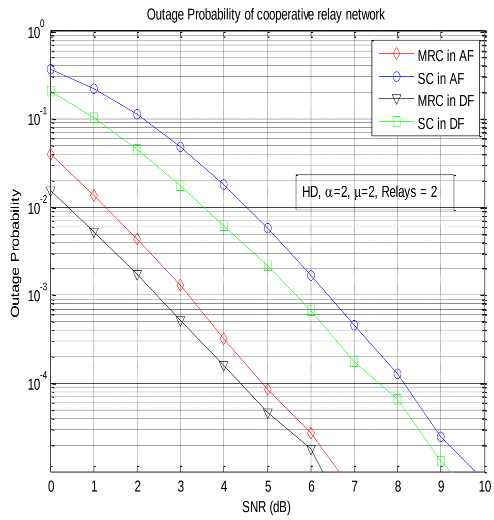

Fig. 6.34. Outage of half-duplex cooperative relaying over α-μ fading channel for α=2, μ=2, Relays=2.

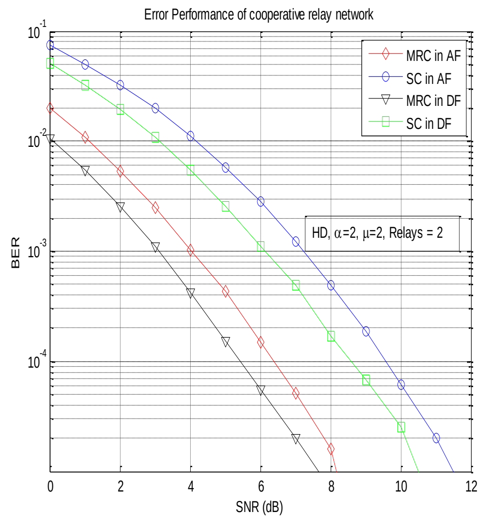

Fig. 6.35. BER of half-duplex cooperative relaying over α-μ fading channel for α=2, μ=2, Relays=2.

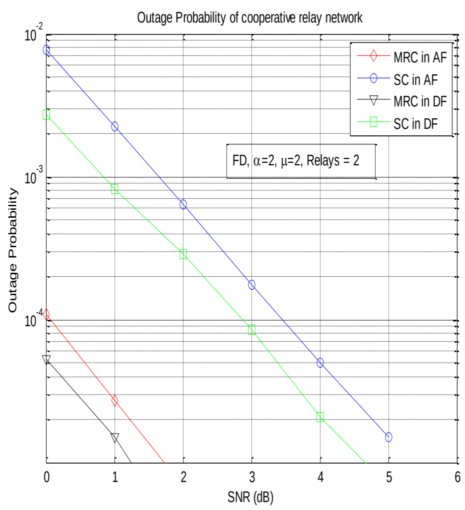

Fig. 6.36. Outage of full-duplex cooperative relaying over α-μ fading channel for α=2, μ=2, Relays=2.

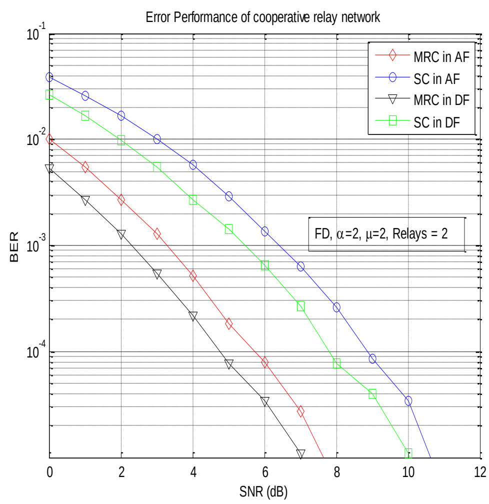

Fig. 6.37.BER of full-duplex cooperative relaying over α-μ fading channel for α=2, μ=2, Relays=2

In fig.6.38 the BER performance for half duplex system with 3 parallel relays, α=2 and μ=2 is shown. It is seen that at 6 dB SNR, the BER for MRC-AF is 5.5×10^{-3}, but for MRC-DF is 2.0×10^{-3} and that for SC-AF is 5×10^{-2} whereas for SC-DF is 3×10^{-2}. From fig. 7 we observe MRC-DF outperforms MRC-AF and similarly SC-DF outperforms SC-AF. Further, error performance for full duplex system with 3 parallel relays, α=2 and μ=2 is shown in fig.6.39, we find the BER at 3 dB SNR for MRC-AF is 3.0×10^{-4}, but for MRC-DF is 9.0×10^{-5} and that for SC-AF is 8×10^{-3} whereas for SC-DF is 3.5×10^{-3}.

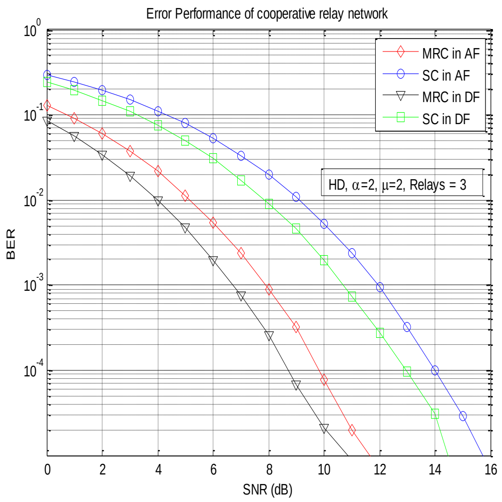

Fig. 6.38.BER of half-duplex cooperative relaying over α-μ fading channel for α=2, μ=2, Relays=3.

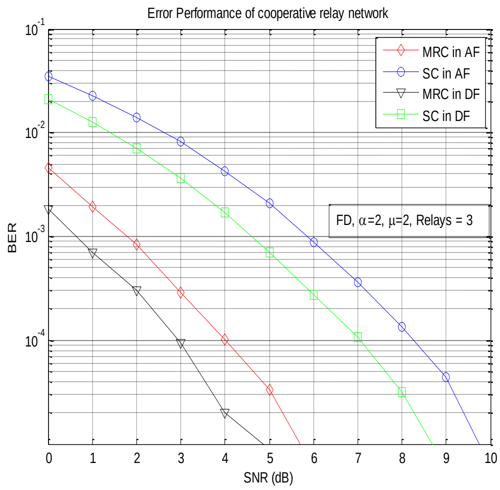

Fig. 6.39.BER of full-duplex cooperative relaying over α-μ fading channel for α=2, μ=2, Relays=3.

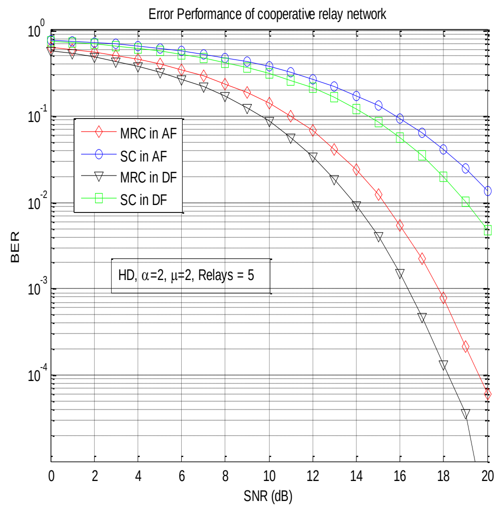

Fig. 6.40.BER of half-duplex cooperative relaying over α-μ fading channel for α=2, μ=2, Relays=5.

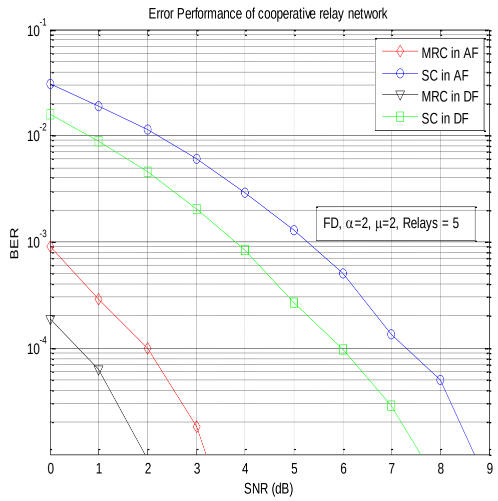

Fig. 6.41.BER of full-duplex cooperative relaying over α-μ fading channel for α=2, μ=2, Relays=5.

In fig.6.40 the BER performance for half duplex system with 5 parallel relays, α=2 and μ=2 is shown. It is seen that at 16 dB SNR, the BER for MRC-AF is 5×10^{-3}, but for MRC-DF is 1.5×10^{-3} and that for SC-AF is 9×10^{-2} whereas for SC-DF is 5.5×10^{-2}. Further, error performance for full duplex system with 5 parallel relays, α=2 and μ=2 is shown in fig.6.41, we find the BER at 1 dB SNR for MRC-AF is 2.8×10^{-4}, but for MRC-DF is 6.0×10^{-5} and that for SC-AF is 2×10^{-2} whereas for SC-DF is 7.5×10^{-3}. The simulation results shows that as expected outage and BER performance for MRC is always better than SC. We also observe that as the number of relays is increased the BER performance deteriorates for HD system and performance improves for FD system.

In this analysis outage probability and BER performance of cooperative communication with multiple parallel relays have been analysed for AF protocol, and DF protocol with MRC and SC over α-μ fading channel for HD and FD wireless systems. It is noticed from comparison of simulation results that MRC-DF outperforms in outage and BER with respect to MRC-AF and similarly SC-DF outperforms SC-AF. The effect of number of parallel relays on outage and BER performance is brought out. It has been analysed that with increase in parallel relays, performance deteriorates for half-duplex system due to increase in time slots but improves in full-duplex system due to proper utilisation of spectrum. The MRC-AF and MRC-DF system outperforms the SC-AF and SC-DF communication systems respectively with large margin, and this difference increases with increase in number of parallel relays. However, for FD relays or a scenario where spectrum efficiency is not an important issue, performance of such systems improves with increment of relays. This result is documented as a research paper ***"Performance Analysis of Cooperative Relaying over α-μ Fading Channel***", and published in International Journal of Innovative Research in Computer and Communication Engineering (IJIRCCE), ISSN 2320-9801, vol.3, iss.8, Aug. 2015, pp.7204-7213. (*DOI: 10.15680/IJIRCCE.2015. 0308115*)

CHAPTER 7

CONCLUSION & FUTURE WORKS

The performance of wireless network in a-μ fading channel has been extensively analysed in this thesis report. Most of the results presented have been already reported (submitted or accepted) in different reputed refereed international journals and peer-reviewed conferences. In our research we have investigated the performance of various communication systems operating in a-μ fading channel. In the following sections we first summarize the research performed in this thesis and then discuss some interesting questions that may be followed up as future work in this area.

7.1 Conclusion:

In chapter 1 introduction of thesis has been presented. Previous works have been discussed, motivation and objective of the research is briefly explained. In chapter 2, we have reviewed multipath fading, parameters and the classification of fading. Wireless fading channel models such as Rayleigh, Rician, Nakagami, Weibull have been revisited. Multipath diversity such as spatial, frequency and time is discussed. Wireless system performance metric parameters such as SNR, outage, BER, amount of fading, AFD and LCR are discussed.

Chapter 3 begins with introduction of generalised fading channels such as κ-μ, η-μ, and α-μ. The α-μ fading channel has been shown as generalised fading channels. Expressions for PDF, CDF, k^{th} moment, SNR, outage, MGF, BER, LCR, average fade duration, and amount of fading is revisited for α-μ fading channel.

In chapter 4 we have analysed the performance of wireless link operating in α-μ fading channel. The α-μ faded random variable have been generated from the algorithm presented in Appendix-B. We have seen that the normalized version of histogram of α-μ distributed random variable perfectly matched with the plot of α-μ PDF. Outage and BER performance of BPSK modulated α-μ fading for different values of α and μ parameters have been shown. Performance of coded α-μ fading is described with brief explanation of error correcting codes, block codes and its generator matrix, cyclic codes, and interleaving techniques. In the analysis, we found that at higher values of α and μ, the BER performance improves significantly due to high power and large number of multipath clusters. Single carrier multi-user CDMA system, with random interleaving are described along with Walsh-Hadamard code. The effect of α and μ parameters on BER performance is brought out by simulation. It has been analyzed that increase in number of users adversely affects the error performance, due to increase in multi user interference.

Performance of α-μ faded wireless link having multiple antennas at receiver have been investigated in chapter 5. Comparative performances of different combining techniques such as SC, SSC, MRC, and EGC with respect to α-μ fading channel for spatial diversity have been evaluated. The effect of α and μ parameters variation on BER and outage is brought out. Various correlation models over α-μ channel is illustrated. Correlated diversity over α-μ channel for outage and BER performance of BPSK modulated signal using SC, SSC, MRC, and EGC combining have been analysed for different correlation coefficients. Performance of correlated α-μ fading channel with varying correlation coefficients for constant and exponential correlation, and different number of antennas in SSC, MRC, and EGC combining have been evaluated. We have observed that performance of MRC is best among all and then the EGC and SC follows the performance subsequently. We have also noticed that the performance of wireless link operating in exponential correlated fading is better than constant correlated fading.

In chapter 6 we have discussed relay based cooperative wireless communication system. Outage and BER performance for amplify-and-forward protocol and decode-and-forward protocol have been plotted with the help of computer simulation. Path loss factor in radio propagation is also briefly discussed. Performance of direct link with AF and DF protocol for two-hop relay based system is analysed in various path loss conditions and we found that AF and DF performance is better than DL in all condition of α, μ and path loss. The relay based system outperforms the direct communication with large margin, and this difference increases with increase of path loss exponent. The performance of regenerative relay system is compared with simulation results for uniform power allocation and optimal power allocation. It has been analyzed that with increase in α and μ parameters, BER and outage performance improves. The relay based system with optimal power allocation outperforms the uniform power allocation. In a multi-hop serial-relay communication system, we have observed that as the number of serial relays are increased, outage and BER for AF and DF deteriorates. The deterioration in AF is more comparative to DF, because in AF with increase in number of relays, noise also gets amplified accordingly. We also notice that performance of full-duplex is better than half-duplex for similar configuration, due to efficient utilization of spectrum, but such system may suffer from loop interference. For HD relay based system, we observe that with increase in relays communication of longer range is achieved but at the cost of degradation in outage and BER performance.

In cooperative communication with multiple parallel relays analysis is carried out for BER performance for AF and DF protocol with MRC and SC over α-μ fading channel for HD and FD wireless systems. It is observed that MRC-DF outperforms in outage and BER with respect to MRC-AF and similarly SC-DF outperforms SC-AF. It has been also shown that with increase in parallel relays diversity order get increased, performance deteriorates for HD system due to unutilised time slots, but improves in FD system due to proper utilisation of spectrum. The MRC-AF and MRC-DF system outperforms the SC-AF and SC-DF communication systems respectively with large margin, and this difference increases with increase in number of parallel relays.

7.2 Suggested Future Work:

Wireless communications have undergone a tremendous development in the last two decades. Due to the popularity of accessing multimedia content on the *world wide web* through wireless devices, fourth generation (4G) and beyond wireless communications are dominated by multimedia applications. Some of the topics pertaining to cooperative communication which may be explored in future are mentioned as below:

7.2.1 Performance analysis of cooperative communication with multi-antenna relay as shown in fig.7.1 may be an important research area.

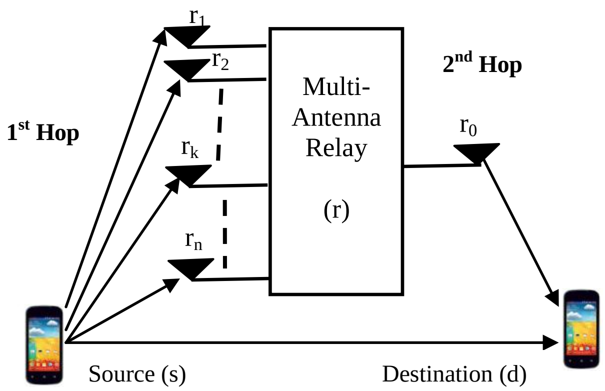

Fig. 7.1. Cooperative Communication system with Multi-Antenna relay

7.2.2 The coded cooperative communication over α-μ is will be also an important area of research because of its efficiency and robustness. Several signals can be simultaneously received in the same time-slot, thus improving the spectrum efficiency of systems.

7.2.3 The cooperative communication exploits the broadcast nature of the wireless medium. The source broadcasts to the relays and each relay transmits the information to the destination. Relay nodes amplify and forward the amount of redundant information to the destination. There may be a point beyond which this redundancy results in diminishing returns in terms of energy efficiency. A recent approach in signal processing area, "compressive sampling" suggests an alternate theory to Nyquist theorem. The central idea in compressive sampling is that the number of samples needed to capture a signal depends primarily on its structural content, rather than its bandwidth. By using nonlinear recovery algorithms, the signals are reconstructed from highly incomplete data. This idea may be applied to the broadcast phase of cooperative communication where the relays can be viewed as data acquisition units in a compressive sampling process. Exploring new protocols and recovery algorithms for the above scheme and analyzing the performance can be an interesting problem that can be pursued in future.

7.2.4 Proceeding on the similar lines of analysis and simulation carried out in this thesis, a research work may be undertaken for cooperative communication over κ-μ and η-μ generalised fading channels.

7.2.5 Considering all the analytical and simulation results presented in this thesis, it will be interesting to design a cooperative communication system that will be exposed to an arbitrary scattering scenario.

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APPENDICES

A. Probability Theory:

In wireless communication the signals are function of time and are of two classes – deterministic and random. Deterministic signals are described by a mathematical function of independent variable time (t). The random signals always have some element of uncertainty associated with it hence its exact value at any given time can't be determined. Therefore we describe the random signal in terms of its average properties such as the average power, spectral distribution, and probability that the signal amplitude exceeds a given value. The probabilistic model used for charactering a random signal is called a ***random process***.

A1. Random Variable (RV) and Random Process: RV is a numeric outcome that results from an experiment. Example: a RV X, where X denotes the number of heads in tossing three coins. For each element of an experiment's sample space, the random variable can take on exactly one value.

- Random Variables are denoted by upper case letters (X).
- Individual outcome of RV is denoted by lower case letter (x).

In above Example the 'number of heads' are outcomes of RV X, can be $x_1 = 0$, $x_2 = 1$, $x_3 = 2$, $x_4 = 3$.

(i) Discrete RV: A RV that can take on only a finite or countable infinite set of outcomes.

(ii) Continuous RV: A RV that can take on any value along a continuum (but may be reported "discretely").

Random Process is a collection (or ensemble) of RVs { $X(s, t)$ } that are function of a real variable, namely time t where s belongs to S (sample space) and t belongs to T (index set). If certain probability distribution or averages do not depend on t, then the random process { $X(t)$ } is called stationary. If all the finite dimensional distributions are invariant of time parameter then the random process is called as strict sense stationary (SSS) random process. If mean is constant and autocorrelation depends on time difference then the random process is called as wide sense stationary (WSS) random process.

A2. Discrete Random Variable:

i) Probability Mass Function or Discrete Probability Distribution or Probability Function: Each discrete RV (X) in a sample space has certain probability of occurrence, which is known as Probability Mass Function $f(x)$, satisfies following condition.

(i) $f(x) \geq 0$, (ii) $\sum f(x) = 1$, (iii) $P(X = x) = f(x)$

For above example the discrete probability distribution function (PDF) will be

$X = x$	0	1	2	3
$P(X = x) = f(x)$	1/8	3/8	3/8	1/8

ii) Cumulative Distribution Function (CDF): The Cumulative Distribution Function F(x) of a discrete RV (X) is defined by

$$F(x) = P(X \leq x) = \sum f(x_i); \quad - \infty < x > \infty, \text{ for all } x_i \leq x$$

For above example the CDF will be

$X = x$	0	1	2	3
$F(x) = \mathrm{P}(\mathrm{X} \leq x)$	1/8	4/8	7/8	8/8

A3. Continuous Random Variable:

i) Continuous Probability Distribution or Probability Density Function (PDF): The PDF of a continuous RV (X) is the function $f(x)$, satisfying following condition.

- $f(x) \geq 0, \quad \text{for all } x$
- $\int_{-\infty}^{\infty} f(x)dx = 1$
- $P(a \leq X \leq b) = \int_{a}^{b} f(x)dx$

ii) Cumulative Distribution Function (CDF): The Cumulative Distribution Function $F(x)$ of a continuous RV (X) is defined by

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)dx, \quad -\infty \leq x \leq \infty, \quad \text{where } f(x) \text{ is PDF}$$

A4. Joint Probability Distribution of *X* and *Y*:

If *X* and *Y* are two discrete random variables, the probability distribution for their simultaneous occurrence can be represented by a function with values *f* (*x*, *y*) for any pair of values (*x*, *y*) within the range of random variables *X* and *Y*. The function *f* (*x*, *y*) is termed as Joint Probability Distribution of *X* and *Y*. The Joint Probability Distribution or Probabi and *Y*, follows following properties:

- $f(x, y) \geq 0;\quad$ for all (x, y)
- $\sum_{x}\sum_{y} f(x, y) = 1$
- $P\left(X = x, Y = y\right) = f(x, y)$

i) Joint Density Function of X and Y: If X and Y are two continuous random variables, the joint density function f (x, y) is a surface lying above the xy plane for any continuous values of (x, y) within the range of random variables X and Y, and follows following properties:

- $f(x, y) \geq 0;\quad$ for all (x, y)
- $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\, dx\, dy = 1$
- $P\left(X, Y \in R\right) = \iint_{R} f(x, y)\, dx\, dy,\quad$ for any region R in the xy plane

ii) Marginal Distribution of *X* and *Y*:

- If x is a discrete RV, $\quad g(x) = \sum_{y} f(x, y)$
- If x is a continuous RV, $\quad g(x) = \int_{-\infty}^{\infty} f(x, y)\, dy$
- If y is a discrete RV, $\quad h(y) = \sum_{x} f(x, y)$
- If y is a continuous RV, $\quad h(y) = \int_{-\infty}^{\infty} f(x, y)\, dx$

iii) Conditional Distribution of *X* and *Y*: If *X* and *Y* are two random variables, discrete or continuous, the conditional distribution of the Random Variable *Y*, given that *X* = *x* is

$$f\left(\frac{y}{x}\right) = \frac{f(x, y)}{g(x)},\quad g(x) \geq 0$$

Similarly, the conditional distribution of the Random Variable *X*, given that *Y* = *y* is

$$f\left(\frac{x}{y}\right) = \frac{f(x, y)}{h(y)},\quad h(y) \geq 0$$

iv) Statistically Independent Random Variables:

If X and Y are two random variables, discrete or continuous with Joint Probability Distribution $f(x, y)$ and Marginal Distribution $g(x)$ and $h(y)$ respectively. The random variables X and Y are said to be Statistically Independent if and only if :

$$f(x, y) = g(x)\, h(y); \text{ For all } x \text{ and } y \text{ within their range.}$$

A5. Correlation Function

The correlation (or inter relationship) between two random variables is a measure of the similarity between the variables.

(I) Auto-Correlation Function:

Auto-Correlation Function is correlation between two random variables when they belong to same Random Process. Consider two Random variables of a Random Process $X(t, s)$: X_{t1} and X_{t2} obtained at two different times at t_1 and t_2. The Auto-correlation function is given as

$$R_{XX}(t_1,\ t_2) = E\,[X_{t_1}\ X_{t_2}] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 f_X(x_1 x_2; t_1 t_2) dx_1 dx_2$$

If $t_1 = t$, and $t_2 - t_1 = \tau$, then the Auto-correlation function will be given as

$$R_{XX}(t,\ t+\tau) = R_{XX}(\tau) = E\,[X_t\ X_{t+\tau}] = \int_{-\infty}^{\infty} x(t)x(t+\tau) f_X(x)dx$$

- Mean square value (Power) = $R_{XX}(0) = E\,[\,X_t^2\,]$
- Even Symmetry; $R_{XX}(\tau) = R_{XX}(-\tau)$

(II) Cross-Correlation Function:

Cross-Correlation Function is correlation between two random variables when they belong to different Random Process. Consider two Random variables X_t of Random Process $X(t, s)$ and Y_t of a Random Process $Y(t, s)$. The cross-correlation function is given as

$$R_{XY}(t_1,\ t_2) = E\,[X_{t_1}\ Y_{t_2}] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 y_2 f_{XY}(x_1 y_2; t_1 t_2) dx_1 dy_2$$

If $t_1 = t$, and $t_2 - t_1 = \tau$, then the cross-correlation function will be given as

$$R_{XY}(t,\ t+\tau) \ = \ R_{XY}(\tau) = E\ [X_t\ Y_{t+\tau}] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1 y_2 f_{XY}(x_1, y_2) dx_1 dy_2$$

- Orthogonal Processes: $R_{XY}(t, t+\tau) = 0$
- Statistically Independent Processes: $R_{XY}(t, t+\tau) = E[X_t]\ E[Y_{t+\tau}]$

A6. Transformation of variables

Papoulis and Pillai (2002) have defined the transformation of random variables. Let X be a continuous RV, with PDF $f_X(x)$, and also $Y=g(X)$, then by change of variable technique, PDF of Y is given as

$$f_Y(y) \ = \left[f_X(x) \ \times \ \frac{dx}{dy} \right]_{x=g^{-1}(y)}$$

A7. Independent and Identically Distributed (i.i.d.) fading channels:

In probability theory and statistics, a sequence or other collection of random variables is independent and identically distributed (i.i.d.) if each random variable has the same probability distribution as the others and all are mutually independent. Thus a fading channel can be called as i.i.d. if signal of each link in the channel has same probability distribution and all links are mutually independent.

Suppose X_1 and X_2 are independent and identically distributed (i.i.d.) continuous random variables. By *independent* it means that

$$P\{X_1 \in A, X_2 \in B\} \ = P\{X_1 \in A\} \times P\{X_2 \in B\}$$

By *identically distributed* it means that X_1 and X_2 each have the same distribution function and therefore the same density function.

B. Analytical and simulated α-μ PDF:

The PDF of fading envelope of α-μ fading channel is defined by eq.(3.33) is given as below:

$$f_R(r) = \frac{\alpha\ \mu^{\mu}\ r^{\alpha\mu\ -\ 1}}{\hat{r}^{\alpha\mu}\ \Gamma(\mu)} \quad \exp\left[-\mu \frac{r^{\alpha}}{\hat{r}^{\alpha}}\right]$$

Algorithm given in Table-B.1 is used for generation of α-μ distributed random variable in the simulation work reported in this thesis.

Table-B.1: Algorithm for generation of α-μ distributed random variable

1. **Procedure *α-μ*** random variable generation
2. $\alpha \leftarrow$ Channel Parameter
3. $\mu \leftarrow$ Channel Parameter
4. x ← Number of random variables
5. Ω ← Mean
6. $\sigma^2 \leftarrow$ Variance
7. H ← zero matrix of order 1×x
8. **for** $i \leftarrow 1$ to μ **do**
9. H = H + matrix of order 1×x having complex Gaussian random variable i.e. X(Ω, σ^2) +j X(Ω, σ^2)
10. end **for**
11. Fading envelope ← H^{α}
12. **end** procedure

Analytical and simulated results for PDF of fading envelope of α-μ fading channel defined by eq.(3.33), are shown for α=7/4 and μ=3 in fig.B.1. It is verified that both the analytical and simulated results are matching.

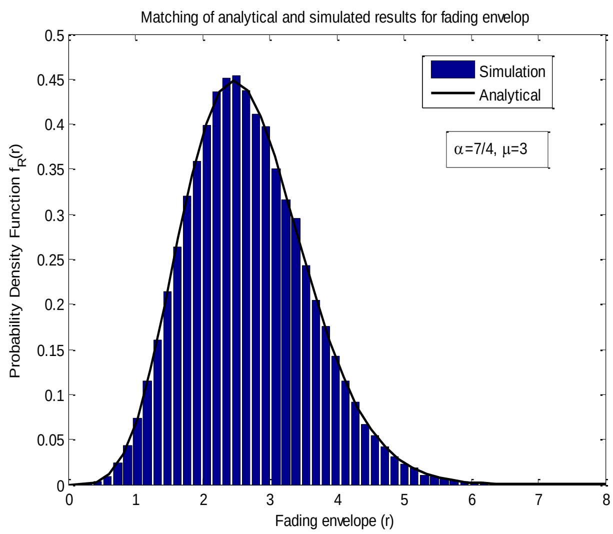

Fig. B.1. Analytical and simulated results of PDF of α-μ fading envelope.

C1. Monte-Carlo simulation

Monto-Carlo Simulation is a problem solving technique used to approximate the probability of certain outcomes by running multiple trial runs, called simulations, using random variables. Monte Carlo simulation is named after the city in Monaco, which is famous for its casino, which have games of chance. These gambling games, like roulette, involve repetitive events exhibit random behavior with known probabilities. Mathematician Stanislaw Ulam is credited with recognizing how computers could make Monte Carlo simulation of complex systems feasible in 1940s. Today, Monte Carlo simulation is perhaps the most commonly used technique, to compute the probability distribution of predicted performance. In this the input uncertainties are propagated (translated) into uncertainties in the results.

In Monte Carlo simulation, the entire system is simulated a large number (e.g., 100000) of times. Each simulation is equally likely, referred to as a realization of the system. For each realization, all of the uncertain parameters are sampled (i.e., a single random value is selected from the specified distribution describing each parameter). The system is then simulated through time (given the particular set of input parameters) such that the performance of the system can be computed. This results is a large number of separate and independent results, each representing a possible "future" for the system (i.e., one possible path the system may follow through time). The results of the independent system realizations are assembled into probability distributions of possible outcomes. As a result, the outputs are not single values, but probability distributions.

C2. Gamma Function:

The Lower Incomplete Gamma function $\gamma(s, x)$ defined by eq.(8.350.1) in Gradshteyn and Ryzhik (2007) as

$$\gamma(s,x) = \int_0^x t^{s-1}\, e^{-t}\, dt$$

Whereas the Complementary or Upper Incomplete Gamma function $\Gamma(s, x)$ is defined by eq.(8.350.2) in Gradshteyn and Ryzhik (2007) as

$$\Gamma(s,x) = \int_x^\infty \mathrm{t}^{s-1}\, e^{-t}\, \mathrm{dt}$$

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